Zero to Monero: First Edition

a technical guide to a private digital currency; for beginners, amateurs, and experts

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Abstract

Cryptography. It may seem like only mathematicians and computer scientists have access to this obscure, esoteric, powerful, elegant topic. In fact, many kinds of cryptography are simple enough that anyone can learn their fundamental concepts.

Many people know cryptography is used to secure communications, whether they be coded letters or private digital interactions. Another application is in so-called cryptocurrencies. These digital moneys use cryptography to ensure, first and foremost, that no piece of money can be duplicated or created at will. To that end, cryptocurrencies typically rely on 'blockchains', creating public, distributed ledgers containing records of currency transactions that can be verified by third parties [49].

It might seem at first glance that transactions need to be sent and stored in plain text format in order to make them publicly verifiable. In fact, it is possible to conceal participants of transactions, as well as the amounts involved, using cryptographic tools that nevertheless allow transactions to be verified and agreed upon by observers [64]. This is exemplified in the cryptocurrency Monero.

We endeavor here to teach anyone who knows basic algebra and simple computer science concepts like the 'bit representation' of a number not only how Monero works at a deep and comprehensive level, but also how useful and beautiful cryptography can be.

For our experienced readers: Monero is a standard one-dimensional distributed acyclic graph (DAG) cryptocurrency blockchain [49] where transactions are based on elliptic curve cryptography using curve Ed25519 [28], transaction inputs are signed with Schnorr-style multilayered linkable spontaneous anonymous group signatures (MLSAG) [61], and output amounts (communicated to recipients via ECDH [33]) are concealed with Pedersen commitments [45] and Schnorr-style Borromean ring signatures [46]. Much of this report is spent explaining these ideas.

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CHAPTER 1

Introduction

In the digital realm it is often trivial to make endless copies of information, with equally endless alterations. For a currency to exist digitally and be widely adopted, its users must believe its supply is strictly limited. A money recipient must be able to verify they are not receiving counterfeit coins, or coins that have already been sent to someone else. To accomplish that without requiring the collaboration of any third party like a central authority, its supply and complete transaction history must be publicly verifiable.

We can use crytographic tools to allow data registered in an easily accessible database - the blockchain - to be virtually immutable and unforgeable, with legitimacy that cannot be disputed by any party.

Cryptocurrencies store transactions in the blockchain, which acts as a public ledger[1] of all the currency operations. Most cryptocurrencies store transactions in clear text, to facilitate verification of transactions by the community of users.

Clearly, an open blockchain defies any basic understanding of privacy, since it virtually *publicizes* the complete transaction histories of its users.

To address the lack of privacy, users of cryptocurrencies such as Bitcoin can obfuscate transactions by using temporary intermediate addresses [50]. However, with appropriate tools it is possible to analyze flows and to a large extent link true senders with receivers [62, 31, 53].

[1] In this context ledger just means a record of all currency creation and exchange events. Specifically, how much money was transferred in each event and to whom.

In contrast, the cryptocurrency Monero (Moe-neh-row) attempts to tackle the issue of privacy by storing only single-use addresses for receipt of funds in the blockchain, and by authenticating the dispersal of funds in each transaction with ring signatures. With these methods there are no effective ways to link receivers or trace the origin of funds [7].

Additionally, transaction amounts in the Monero blockchain are concealed behind cryptographic constructions, rendering currency flows opaque.

The result is a cryptocurrency with a high level of privacy.

1.1 Objectives

Monero is a cryptocurrency of recent creation [63, 10], yet it displays a steady growth in popularity[2]. Unfortunately, there is little comprehensive documentation describing the mechanisms it uses. Even worse, important parts of its theoretical framework have been published in non peer-reviewed papers which are incomplete and/or contain errors. For significant parts of the theoretical framework of Monero, only the source code is reliable as a source of information.

Moreover, for those without a background in mathematics, learning the basics of elliptic curve cryptography, which Monero uses extensively, can be a haphazard and frustrating endeavor.[3]

We intend to palliate this situation by introducing the fundamental concepts necessary to understand elliptic curve cryptography, reviewing algorithms and cryptographic schemes, and collecting in-depth information about Monero's inner workings.

To provide the best experience for our readers, we have taken care to build a constructive, step-by-step description of the Monero cryptocurrency.

In the first edition of this report we have centered our attention on version 7 of the Monero protocol, corresponding to version 0.12.x.x of the Monero software suite. All transaction related mechanisms described here belong to those versions. Deprecated transaction schemes have not been explored to any extent, even if they may be partially supported for backward compatibility.

1.2 Readership

We anticipate many readers will encounter this report with little to no understanding of discrete mathematics, algebraic structures, cryptography, and blockchains. We have tried to be thorough enough that laymen from all perspectives may learn Monero without needing external research.

[2] As of June 14th, 2018, Monero occupies the 14th position as regards market capitalization; see https://coinmarketcap.com/

[3] A previous attempt to explain how Monero works did not elucidate elliptic curve cryptography, was incomplete, and is now almost four years outdated: [52].

We have purposefully omitted, or delegated to footnotes, some mathematical technicalities, when they would be in the way of clarity. We have also omitted concrete implementation details where we thought they were not essential. Our objective has been to present the subject half-way between mathematical cryptography and computer programming, aiming at completeness and conceptual clarity.

1.3 Origins of the Monero cryptocurrency

The cryptocurrency Monero, originally known as BitMonero, was created in April, 2014 as a derivative of the proof-of-concept currency CryptoNote [63]. Monero means 'money' in the language Esperanto, and its plural form is Moneroj (Moe-neh-rowje, similar to Moneros but using the -ge from orange).

CryptoNote is a cryptocurrency devised by various individuals. A landmark whitepaper describing it was published under the pseudonym of Nicolas van Saberhagen in October, 2013 [64]. It offered receiver anonymity through the use of one-time addresses, and sender ambiguity by means of ring signatures.

Since its inception, Monero has further strengthened its privacy aspects by implementing amount hiding, as described by Greg Maxwell (among others) in [46] and integrated into ring signatures based on Shen Noether's recommendations in [61].

1.4 Outline

As mentioned, our aim is to deliver a self-contained and step-by-step description of the Monero cryptocurrency. This report has been structured to fulfill this objective, leading the reader through all parts of the currency's inner workings.

In our quest for comprehensiveness, we have chosen to present all the basic elements of cryptography needed to understand the complexities of Monero, and their mathematical antecedents. In Chapter 2 we develop essential aspects of elliptic curve cryptography.

Chapter 3 outlines the ring signature algorithms that will be applied to achieve confidential transactions.

In Chapter 4 we introduce the cryptographic mechanisms used to conceal amounts.

With all the components in place, we explain the transaction schemes used in Monero in Chapter 5.

We unfold the Monero blockchain in Chapter 6.

While not essential to the operation of Monero, there is a lot of utility in multisignatures that allow multiple people to send and receive money collaboratively. Simple multisignatures (N-of-N and (N-1)-of-N, with naive key aggregation) are currently available in the source code. However, the Monero Research Lab is currently writing a technical paper on the subject, with several improvements and a security proof 'in the works'. Readers can find our writeup (subject to change) on multisignatures here: https://github.com/UkoeHB/Monero-RCT-report, or look forward to its inclusion in future editions.

Appendices A and B explain the structure of sample transactions from the blockchain. Appendix C explains the structure of blocks (including block headers and miner transactions) in Monero's blockchain, while Appendix D brings our report to a close by explaining the structure of Monero's genesis block. These provide a connection between the theoretical elements described in earlier sections with their real-life implementation.

We use margin notes to indicate where Monero implementation details can be found in the source code.[4] There is usually a file path, such as src/ringct/rctSigs.cpp, and a function, such as genBorromean(). Note: '-' indicates split text, such as crypto- note → cryptonote.

Isn't this useful?

1.5 Disclaimer

All signature schemes, applications of elliptic curves, and Monero implementation details should be considered descriptive only. Readers considering serious practical applications (as opposed to a hobbyist's explorations) should consult original sources and technical specifications (which we have cited where possible). Signature schemes need well-vetted security proofs, and Monero implementation details can be found in Monero's source code. In particular, as a common saying goes, 'don't roll your own crypto'. Code implementing cryptographic primitives should be well reviewed by experts, and have a long history of solid performance.

1.6 History of Zero to Monero

Zero to Monero is an expansion of Kurt Alonso's master's thesis 'Monero - Privacy in the Blockchain' [24].

[4] These margin notes are accurate for version 0.12.x.x of the Monero software suite, but may gradually become inaccurate as the code base is constantly changing. However, the code is stored in a git repository (https://github.com/monero-project/monero), so a complete history of changes is available.

CHAPTER 2

Basic Concepts

2.1 A few words about notation

One focal objective of this report was to collect, review, correct and homogenize all existing information concerning the inner workings of the Monero cryptocurrency. And, at the same time supply all the necessary details to present the material in a constructive and single-threaded manner.

An important instrument to achieve this was to settle for a number of notational conventions. Among others, we have used:

- lower case letters to denote simple values, integers, strings, bit representations, etc
- upper case letters to denote curve points and complicated constructs

For items with a special meaning, we have tried to use as much as possible the same symbols throughout the document. For instance, a curve generator is always denoted by G, its order is l, private/public keys are denoted whenever possible by k/K respectively, etc.

Beyond that, we have aimed at being *conceptual* in our presentation of algorithms and schemes. A reader with a computer science background may feel we have neglected questions like the bit representation of items, or, in some cases, how to carry out concrete operations. Moreover, students of mathematics may find we disregarded explanations of abstract algebra.

However, we don't see this as a loss. A simple object such as an integer or a string can always be represented by a bit string. So-called *endianness* is rarely relevant, and is mostly a matter of convention for our algorithms.

Elliptic curve points are normally denoted by pairs (x, y), and can therefore be represented with two integers. However, in the world of cryptography it is common to apply *point compression* techniques, which allow representing a point using only the space of one coordinate. For our conceptual approach it is often accessory whether point compression is used or not, but most of the time it is implicitly assumed.

We have also used cryptographic hash functions freely without specifying any concrete algorithms. In the case of Monero it will typically be a *Keccak*[1] variant, but if not explicitly mentioned then it is not important to the theory.

src/crypto/
keccak.c

A cryptographic hash function (henceforth simply 'hash function', or 'hash') takes in some message $\mathfrak{m}$ of arbitrary length and returns a hash h (or *message digest*) of fixed length, with each possible output equiprobable for a given input. Cryptographic hash functions are difficult to reverse, have an interesting feature known as the *large avalanche effect* which can cause very similar messages to produce very dissimilar hashes, and it is hard to find two messages with the same message digest.

Hash functions will be applied to integers, strings, curve points, or combinations of these objects. These occurrences should be interpreted as hashes of bit representations, or the concatenation of such representations. Depending on context, the result of a hash will be numeric, a bit string, or even a curve point. Further details in this respect will be given as needed.

2.2 Modular arithmetic

Most modern cryptography begins with modular arithmetic, which in turn begins with the modulus operation (denoted 'mod'). We only care about the positive modulus, which always returns a positive integer.

The positive modulus is here defined for $a \pmod{b} = c$ as $a = bx + c$, where $0 \leq c < b$ and x is a signed integer which gets discarded (note that b is an integer bigger than zero).

Let's imagine a number line. We stand at point a, then walk toward zero with each step $= b$ until we reach an integer ≥ 0 and $< b$. That is c. For example, $4 \pmod{3} = 1$, $-5 \pmod{4} = 3$, and so on. Readers may recognize c is similar to the 'remainder' after dividing (a/b).

Note that, if $a \leq n$, $-a \pmod{n}$ is the same as $n - a$.

[1] The Keccak hashing algorithm forms the basis for the NIST standard *SHA-3* [20].

2.2.1 Modular addition and multiplication

In computer science it is important to avoid large numbers when doing modular arithmetic. For example, if we have $29 + 87 \pmod{99}$ and we aren't allowed variables with three or more digits (such as $116 = 29 + 87$), then we can't compute $116 \pmod{99} = 17$ directly.

To perform $c = a + b \pmod{n}$, where a and b are each less than the modulus n, we can do this:

- Compute $x = n - a$. If $x > b$ then $c = a + b$, otherwise $c = b - x$.

We can use modular addition to achieve modular multiplication ($a * b \pmod{n} = c$) with an algorithm called 'double-and-add'. Let us demonstrate by example. Say we want to perform $7 * 8 \pmod{9} = 2$. This is the same as

$$7 * 8 = 8 + 8 + 8 + 8 + 8 + 8 + 8 \pmod{9}$$

Now break this into groups of two.

$$(8 + 8) + (8 + 8) + (8 + 8) + 8$$

And again, by groups of two.

$$[(8 + 8) + (8 + 8)] + (8 + 8) + 8$$

The total number of + point operations falls from 6 to 4 because we only need to find $(8+8)$ once.

Double-and-add is implemented by converting the first number (the 'multiplicand' a) to binary (in our example, $7 \rightarrow [0111]$), then going through the binary array and doubling and adding.

Let's make an array $A = [0111]$ and index it 0,1,2,3. A[0] = 0 is the first element of A and is the most significant bit. We set a result variable to be initially $r = 0$, and set a sum variable to be initially $s = 8$ (more generally, we start with $s = b$). We follow this algorithm:

1. Iterate through: $i = (A_{size} - 1, ..., 0)$
   (a) If A[i] == 1, then $r = r + s \pmod{n}$.
   (b) Compute $s = s + s \pmod{n}$.
2. Use the final r: $c = r$.

In our example, this sequence appears:

1. $i = 3$
   (a) A[3] = 1, so $r = 0 + 8 \pmod{9} = 8$
   (b) $s = 8 + 8 \pmod{9} = 7$
2. $i = 2$

   (a) A[2] $= 1$, so $r = 8 + 7 \pmod 9 = 6$
   (b) $s = 7 + 7 \pmod 9 = 5$

3. $i = 1$
   (a) A[1] $= 1$, so $r = 6 + 5 \pmod 9 = 2$
   (b) $s = 5 + 5 \pmod 9 = 1$

4. $i = 0$
   (a) A[0] $= 0$, so r stays the same
   (b) $s = 1 + 1 \pmod 9 = 2$

5. $r = 2$ is the result

2.2.2 Modular exponentiation

It's obvious that $8^7 \pmod 9$ is the same as $8*8*8*8*8*8*8 \pmod 9$. Just like double-and-add, we can do square-and-multiply. For $a^e \pmod n$:

1. Define $e_{scalar} \rightarrow e_{binary}$; $A = [e_{binary}]$; $r = 1$; $m = a$
2. Iterate through: $i = (A_{size} - 1, ..., 0)$
   (a) If A[i] == 1, then $r = r * m \pmod n$.
   (b) Compute $m = m * m \pmod n$.
3. Use the final r as result.

2.2.3 Modular multiplicative inverse

Sometimes we need to calculate $1/b \pmod n$, or in other words $b^{-1} \pmod n$.

In the equation $a \equiv b \pmod n$, a is *congruent* to $b \pmod n$, which just means $a \pmod n = b \pmod n$.

The modular multiplicative inverse is defined as an integer c such that, for $c = a^{-1} \pmod n$, $ac \equiv 1 \pmod n$ for $0 \leq c < n$ and for a and n relatively prime. Relatively prime just means they don't share any divisors except 1. We could also say the fraction a/n can't be reduced/simplified.[2]

[2] We want to point out the modular multiplicative inverse has a rule stating:
If $ac \equiv b \pmod n$ with a and n relatively prime, the solution to this linear congruence is given by $c = a^{-1}b \pmod n$.[9]
This means we can do $c = a^{-1}b \pmod n \rightarrow ca \equiv b \pmod n \rightarrow a \equiv c^{-1}b \pmod n$.

We can use square-and-multiply to compute the modular multiplicative inverse when n is a prime number because of *Fermats' little theorem*:

$$\begin{aligned} a^{n-1} &\equiv 1 \pmod{n} \\ a * a^{n-2} &\equiv 1 \pmod{n} \\ c \equiv a^{n-2} &\equiv a^{-1} \pmod{n} \end{aligned}$$

More generally (and more rapidly), the so-called extended Euclidean algorithm finds c like this (short and sweet just to expose it):

1. Set $r = 0$; $r' = 1$; $q = n$; $q' = a$.
2. While $r' \neq 0$, iterate through these steps:
   - $quotient = integer(q/q')$
   - $(r, r') = (r', r - quotient * r')$ [3]
   - $(q, q') = (q', q - quotient * q')$
3. If $q \leq 1$ then (if $r < 0$ then $c = r + n$, else $c = r$), else no solution.

2.2.4 Modular equations

Suppose we have an equation $c = 3 * 4 * 5 \pmod{9}$. Computing this is straightforward. Given some operation $\circ$ (for example, $\circ = *$) between two variables A and B:

$$(A \circ B) \pmod{n} = [A \pmod{n}] \circ [B \pmod{n}] \pmod{n}$$

In our example, we set $A = 3 * 4$, $B = 5$, and $n = 9$:

$$\begin{aligned} (3 * 4 * 5) \pmod{9} &= [3 * 4 \pmod{9}] * [5 \pmod{9}] \pmod{9} \\ &= [3] * [5] \pmod{9} \\ &= 6 \end{aligned}$$

Now we have a way to do modular subtraction.

$$\begin{aligned} A - B \pmod{C} &\rightarrow A + (-B) \pmod{n} \\ &\rightarrow [A \pmod{n}] + [-B \pmod{n}] \pmod{n} \end{aligned}$$

The same principle would apply to something like $x = (a - b * c * d)^{-1}(e * f + g^h) \pmod{n}$.

[3] We use r and r' from the previous round to compute the current round's r and r' at the same time.

2.3 Elliptic curve cryptography

2.3.1 What are elliptic curves?

A finite field $\mathbb{F}_q$, where q is a prime number greater than 3, is the field formed by the set $\{0, 1, 2, ..., q-1\}$. Addition and multiplication $(+, \cdot)$ and negation $(-)$ are calculated $\pmod q$. fe: field element

"Calculated $\pmod q$" means $\pmod q$ is performed on any instance of an arithmetic operation between two field elements, or negation of a single field element. For example, given a prime field $\mathbb{F}_p$ with $p = 29$, $17 + 20 = 8$ because $37 \pmod{29} = 8$. Also, $-13 = -13 \pmod{29} = 16$.

Typically, an elliptic curve with a given (a, b) pair is defined as the set of points (x, y) satisfying a *Weierstraß* equation [37]:[4]

$$y^2 = x^3 + ax + b \quad \text{where} \quad a, b, x, y \in \mathbb{F}_q$$

The cryptocurrency Monero uses a special curve belonging to the category of so-called *Twisted Edwards* curves [28], which are commonly expressed as (for a given (a, d) pair):

$$ax^2 + y^2 = 1 + dx^2y^2 \quad \text{where} \quad a, d, x, y \in \mathbb{F}_q$$

In what follows we will prefer this second form. The advantage it offers over the previously mentioned Weierstraß form is that basic cryptographic primitives require fewer arithmetic operations, resulting in faster cryptographic algorithms (see Bernstein *et al.* in [30] for details).

Let $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ be two points belonging to a Twisted Edwards elliptic curve (henceforth known simply as an EC). We define addition on points by defining $P_1 + P_2 = (x_1, y_1) + (x_2, y_2)$ as the point $P_3 = (x_3, y_3)$ where

$$x_3 = \frac{x_1y_2 + y_1x_2}{1 + dx_1x_2y_1y_2} \pmod q$$
$$y_3 = \frac{y_1y_2 - ax_1x_2}{1 - dx_1x_2y_1y_2} \pmod q$$

These formulas for addition also apply for point doubling; that is, when $P_1 = P_2$. To subtract a point, invert its coordinates over the y-axis, $(x, y) \to (-x, y)$ [28], and use point addition. Whenever a 'negative' element $-x$ of $\mathbb{F}_q$ appears in this report, it is really $-x \pmod q$.

It turns out that elliptic curves have *abelian group* structure[5] under the addition operation described. Each time two curve points are added together, P_3 is a point on the 'original' elliptic curve, or in other words all $x_3, y_3 \in \mathbb{F}_q$.

Each point P in EC can generate a subgroup of order (size) u out of some of the other points in EC using multiples of itself. For example, some point P's subgroup might have order 5 and

[4] Notation: the phrase $a \in \mathbb{F}$ means a is some element in the field $\mathbb{F}$.

[5] A concise definition of this notion can be found under `https://brilliant.org/wiki/abelian-group/`

contain the points $(0, P, 2P, 3P, 4P)$, each of which is in EC. At $5P$ the so-called *point-at-infinity* appears, which is like the 'zero' position on an EC, and has coordinates $(0, 1)$.

Conveniently, $5P + P = P$. This means the subgroup is *cyclic*[6]. All P in EC generate a cyclic subgroup. If P generates a subgroup whose order is prime then all the included points (except for the point-at-infinity) generate that same subgroup. In our example, take multiples of point $2P$:

$$2P, 4P, 6P, 8P, 10P \to 2P, 4P, 1P, 3P, 0$$

Another example: a subgroup with order 6 $(0, P, 2P, 3P, 4P, 5P)$. Multiples of point $2P$:

$$2P, 4P, 6P, 8P, 10P, 12P \to 2P, 4P, 0, 2P, 4P, 0$$

Here $2P$ has order 3. Since 6 is not prime, not all of its member points recreate the original subgroup.

Each EC has an order N equal to the total number of points in the curve (including the point-at-infinity), and the orders of all subgroups generated by points are divisors of N (by *Lagrange's theorem*). We can imagine a set of all EC points $\{0, P_1, ..., P_{N-1}\}$. If N isn't prime, some points will make subgroups with orders equal to divisors of N.

To find the order, u, of any given point P's subgroup:

1. Find N (e.g. use *Schoof's algorithm*).
2. Find all the divisors of N.
3. For every divisor n of N, compute nP.
4. The smallest n such that $nP = 0$ is the order u of the subgroup.

ECs selected for cryptography typically have $N = hl$, where l is some sufficiently large prime number (such as 160 bits) and h is the so-called *cofactor* which could be as small as 1 or 2.[7] One point in the subgroup of size l is usually selected to be the generator G as a convention. For every other point P in that subgroup there exists an integer $0 < n \leq l$ satisfying $P = nG$.

ge: group element

Let's expand our understanding. Say there is a point P' with order N, where $N = hl$. Any other point P_i can be found with some integer n_i such that $P_i = n_iP'$. If $P_1 = n_1P'$ has order l, any $P_2 = n_2P'$ with order l must be in the same subgroup as P_1 because $lP_1 = 0 = lP_2$, and if $l(n_1P') \equiv l(n_2P') \equiv NP' = 0$, then n_1 & n_2 must both be multiples of h. Since $N = hl$, there are only l multiples of h, implying only one subgroup of size l is possible.

In other words, the subgroup formed by multiples of (hP') always contains P_1 and P_2. Furthermore, $h(n'P') = 0$ when n' is a multiple of l, and there are only h variations of n' $\pmod{N}$ (including the point at infinity for $n' = hl$) because when $n' = hl$ it cycles back to 0: $hlP' = 0$. So, there are only h points P in EC where hP will equal 0.

[6] Cyclic subgroup means, for P's subgroup with order u, and with any integer n, $nP = [n \pmod{u}]P$.
[7] EC with small cofactors allow relatively faster point addition, etc. [28].

A similar argument could be applied to any subgroup of size u: any two points P_1 and P_2 with order u are in the same subgroup, which is composed of multiples of $(N/u)P'$.

With this new understanding it is clear we can use the following algorithm to find points in the subgroup of order l:

1. Find N of the elliptic curve EC, choose subgroup order l, compute $h = N/l$.
2. Choose a random point P' in EC.
3. Compute $P = hP'$.
4. If $P = 0$ return to step 2, else P is in the subgroup of order l.

Calculating the scalar product between any integer n and any point P, nP, is not difficult, whereas finding n such that $P_1 = nP_2$ is known to be computationally hard. By analogy to modular arithmetic, this is often called the *discrete logarithm problem* (DLP). In other words, scalar multiplication can be seen as a *one-way function*, which paves the way for using elliptic curves for cryptography. sc: scalar

The scalar product nP is equivalent to $(((P + P) + P) \dots)$. Though not always the most efficient approach, we can use double-and-add like in Section 2.2.1. To get the sum $R = nP$, remember we use the + point operation discussed in Section 2.3.1.

```
1. Define $n_{scalar} \rightarrow n_{binary}$; $A = [n_{binary}]$; $R = 0$, the point-at-infinity; $S = P$
2. Iterate through: $i = (A_{size} - 1, ..., 0)$
   (a) If A[i] == 1, then R += S.
   (b) Compute S += S.
3. Use the final R as result.
```

2.3.2 Public key cryptography with elliptic curves

Public key cryptography algorithms can be devised in a way analogous to modular arithmetic.

Let k be a randomly selected number satisfying $0 \leq k < l$, and call it a *private key*.[8] Calculate the corresponding *public key* K (an EC point) with the scalar product $kG = K$.

Due to the *discrete logarithm problem* (DLP) we can not easily deduce k from K alone. This property allows us to use the values (k, K) in common public key cryptography algorithms.

[8] The private key is sometimes known as a *secret key*. This lets us abbreviate: pk = public key, sk = secret key.

2.3.3 Diffie-Hellman key exchange with elliptic curves

A basic *Diffie-Hellman* [33] exchange of a shared secret between *Alice* and *Bob* could take place in the following manner:

1. Alice and Bob generate their own private/public keys (k_A, K_A) and (k_B, K_B). Both publish or exchange their public keys, and keep the private keys for themselves.
2. Clearly, it holds that
$$S = k_A K_B = k_A k_B G = k_B k_A G = k_B K_A$$
Alice could privately calculate $S = k_A K_B$, and Bob $S = k_B K_A$, allowing them to use this single value as a shared secret.

   For example, if Alice has a message m to send Bob, she could hash the shared secret $h = \mathcal{H}(S)$, compute $x = m + h$, and send x to Bob. Bob computes $h' = \mathcal{H}(S)$, calculates $m = x - h'$, and learns m.

An external observer would not be able to easily calculate the shared secret due to the DLP, which prevents them from finding k_A or k_B. Since the output of hash functions is 'random', the message m is information-theoretic secure from adversaries who know x, K_A, and K_B.[9]

2.3.4 Schnorr signatures and the Fiat-Shamir transform

In 1989 Charles Schnorr published a now famous interactive authentication protocol [58], generalized by Maurer in 2009 [44], that allowed someone to prove they know the private key k of a given public key K without revealing any information about it [46]. It goes something like this:

1. The prover generates a random integer $\alpha \in_R \mathbb{Z}_l$,[10] computes αG, and sends αG to the verifier.
2. The verifier generates a random *challenge* $c \in_R \mathbb{Z}_l$ and sends c to the prover.
3. The prover computes the *response* $r = \alpha + c * k$ and sends r to the verifier.
4. The verifier computes $R = rG$ and $R' = \alpha G + c * K$, and checks $R \overset{?}{=} R'$.

The verifier can compute $R' = \alpha G + c * K$ before the prover, so providing c is like saying "I challenge you to respond with the discrete logarithm of R'." A challenge the prover can only overcome by knowing k (except with negligible probability).

[9] A cryptosystem with information-theoretic security is one where even an adversary with infinite computing power could not break it, because they simply wouldn't have enough information.

[10] Notation: the R in $\alpha \in_R \mathbb{Z}_l$ means α is randomly selected from $\{0, 1, 2, ..., l-1\}$. In other words, $\mathbb{Z}_l$ is all integers $\pmod{l}$.

If α was chosen randomly by the prover then r is randomly distributed [59] and k is information-theoretically secure. However, if the prover reuses α to prove his knowledge of k, anyone who knows both challenges in $r = \alpha + c * k$ and $r' = \alpha + c' * k$ can compute k (two equations, two unknowns).[11]

$$k = \frac{r - r'}{c - c'}$$

If the prover knew c from the beginning (i.e. if the verifier secretly gave it to her), she could generate a random response r and compute $\alpha G = rG - cK$. When she later sends r to the verifier, she 'proves' knowledge of k without ever having known it. Someone observing the transcript of events between prover and verifier would be none the wiser. The scheme is not *publicly verifiable.* [46]

In his role as challenger, the verifier spits out a random number after receiving αG, making him equivalent to a *random function.* Random functions, such as hash functions, are known as random oracles because computing one is like requesting a random number from someone [46].[12]

Using a hash function, instead of the verifier, to generate challenges is known as a *Fiat-Shamir transform* [34], because it makes an interactive proof non-interactive and publicly verifiable [46].

Non-interactive proof

1. Generate random number $\alpha \in_R \mathbb{Z}_l$, and compute αG.
2. Calculate the challenge using a cryptographically secure hash function, $c = \mathcal{H}([\alpha G])$.
3. Define the response $r = \alpha + c * k$.
4. Publish the proof pair $(\alpha G, r)$.

Verification

1. Calculate the challenge: $c' = \mathcal{H}([\alpha G])$.
2. Compute $R = rG$ and $R' = \alpha G + c' * K$.
3. If $R = R'$ then the prover must know k (except with negligible probability).

[11] If the prover is a computer, you could imagine someone 'cloning'/copying the computer after it generates α, then presenting each copy with a different challenge.

[12] More generally, "[i]n cryptography... an oracle is any system which can give some extra information on a system, which otherwise would not be available." [2]

Why it works

$$\begin{aligned} rG &= (\alpha + c * k)G \\ &= (\alpha G) + (c * kG) \\ &= \alpha G + c * K \\ R &= R' \end{aligned}$$

An important part of any proof/signature scheme is the resources required to verify them. This includes space to store proofs, and time spent verifying. In this scheme we store one EC point and one integer, and need to know the public key - another EC point.

Since hash functions are comparatively fast to compute, verification time is mostly a function of elliptic curve operations. For this proof scheme there are (based on operations available in Monero's code): src/ringct/rctOps.cpp

PA Point addition for some points A, B: $A + B$ [1]
PS Point subtraction for some points A, B : $A - B$ [0]
KBSM Known-base scalar multiplications for some integer a: aG [1]
VBSM Variable-base scalar multiplications for some integer a, and point P: aP [1]
SVBA Single-variable-base addition for some integer a, and point B: $aG + B$ [0]
DVBA Double-variable-base addition for some integers a, b, and point B: $aG + bB$ [0]
VBA Variable-base additions for some integers a, b, and points A, B: $aA + bB$ [0]

2.3.5 Signing messages

Typically, a cryptographic signature is performed on a cryptographic hash of a message rather than the message itself, which facilitates signing messages of varying size. However, in this report we will loosely use the term *message*, and its symbol $\mathfrak{m}$, to refer to the message properly speaking and/or its hash value, unless specified.

Signing messages is a staple of Internet security. One common signature scheme is called ECDSA. See [39], ANSI X9.62, and [37].

The signature scheme we present here is an alternative formulation of the transformed Schnorr proof from before. Thinking of signatures in this way prepares us for exploring ring signatures in the next chapter.

Signature

Assume that Alice has the private/public key pair (k_A, K_A). To unequivocally sign an arbitrary message $\mathfrak{m}$, she could execute the following steps:

1. Generate random number $\alpha \in_R \mathbb{Z}_l$, and compute αG.

2. Calculate the challenge using a cryptographically secure hash function, $c = \mathcal{H}(\mathfrak{m}, [\alpha G])$.
3. Define the response r such that $\alpha = r + c * k_A$. In other words, $r = \alpha - c * k_A$.
4. Publish the signature (c, r).

Verification

Any third party who knows the EC domain parameters D (specifying which elliptic curve was used), the signature (c, r) and the signing method, $\mathfrak{m}$ and the hash function, and K_A can verify the signature:

1. Calculate the challenge: $c' = \mathcal{H}(\mathfrak{m}, [rG + c * K_A])$.
2. If $c = c'$ then the signature passes.

In this signature scheme we store two scalars, need one public EC key, and verify with: src/ringct/rctOps.cpp

DVBA Double-variable-base addition for some integers a, b, and point B: $aG + bB$ [1]

Why it works

This stems from the fact that

$$\begin{aligned} rG &= (\alpha - c * k_A)G \\ &= \alpha G - c * K_A \\ \alpha G &= rG + c * K_A \\ \mathcal{H}_n(\mathfrak{m}, [\alpha G]) &= \mathcal{H}_n(\mathfrak{m}, [rG + c * K_A]) \\ c &= c' \end{aligned}$$

Therefore the owner of k_A (Alice) created (c, r) for $\mathfrak{m}$: she signed the message. The probability someone else, a forger without k_A, could have made (c, r) is negligible.

2.4 Curve Ed25519

Monero uses a particular Twisted Edwards elliptic curve for cryptographic operations, *Ed25519*, the *birational equivalent*[13] of the Montgomery curve *Curve25519*.

Both Curve25519 and Ed25519 were released by Bernstein *et al.* [28, 29, 30].[14]

[13] Without giving further details, birational equivalence can be thought of as an isomorphism expressible using rational terms.

[14] Dr. Bernstein also developed an encryption scheme known as ChaCha [27, 65], which the primary Monero implementation uses to encrypt certain sensitive information related to users' wallets. src/wallet/ringdb.cpp

The curve is defined over the prime field $\mathbb{F}_{2^{255}-19}$ (i.e. $q = 2^{255} - 19$) by means of the following equation:

$$-x^2 + y^2 = 1 - \frac{121665}{121666} x^2 y^2$$

This curve addresses many concerns raised by the cryptography community. It is well known that NIST[15] standard algorithms have issues. For example, it has recently become clear the random number generation algorithm PNRG is flawed and contains a potential backdoor [36]. Seen from a broader perspective, standardization authorities like NIST lead to a cryptographic monoculture, introducing a point of centralization. This was illustrated when the NSA used its influence over NIST to weaken an international cryptographic standard [14].

Curve Ed25519 is not subject to any patents (see [42] for a discussion on this subject), and the team behind it has developed and adapted basic cryptographic algorithms with efficiency in mind [30].

src/crypto/crypto_ops_builder/

Twisted Edwards curves have order expressable as $N = 2^c l$, where l is a prime number and c a positive integer. In the case of curve Ed25519, its order is a 76 digit number (l is 253 bits):[16]

$$2^3 \cdot 7237005577332262213973186563042994240857116359379907606001950938285454250989$$

2.4.1 Binary representation

Elements of $\mathbb{F}_{2^{255}-19}$ are encoded as 256-bit integers. In other words, they can be represented using 32 bytes. Since each element only requires 255 bits, the most significant bit is always zero.

Consequently, any point in Ed25519 could be expressed using 64 bytes. By applying *point compression* techniques, described here below, however, it is possible to reduce this amount by half, to 32 bytes.

2.4.2 Point compression

The Ed25519 curve has the property that its points can be easily compressed, so that representing a point will consume only the space of one coordinate. We will not delve into the mathematics necessary to justify this, but we can give a brief insight into how it works [57]. Point compression for the Ed25519 curve was first described in [29], while the concept was first introduced in [48].

This point compression scheme follows from a transformation of the Twisted Edwards curve equation (assuming $a = -1$, which is true for Monero): $x^2 = (y^2 - 1)/(dy^2 + 1)$,[17] which indicates there are two possible x values (+ or −) for each y. Field elements x and y are calculated $\pmod q$, so there are no actual negative values. However, taking $\pmod q$ of $-x$ will change the value between

[15] National Institute of Standards and Technology, https://www.nist.gov/

[16] This means private EC keys in Ed25519 are 253 bits.

[17] Here $d = -\frac{121665}{121666}$.

odd and even since q is odd. For example: $3 \pmod 5 = 3$, $-3 \pmod 5 = 2$. In other words, the field elements x and $-x$ have different odd/even assignments.

If we have a curve point and know its x is even, but given its y value the transformed curve equation outputs an odd number, then we know negating that number will give us the right x. One bit can convey this information, and conveniently the y coordinate has an extra bit.

src/crypto/crypto_ops_builder/ref10CommentedCombined/ge_to-bytes.c

Assume we want to compress a point (x, y).

Encoding We set the most significant bit of y to 0 if x is even, and 1 if it is odd. The resulting value y' will represent the curve point.

Decoding

ge_from-bytes.c

1. Retrieve the compressed point y', then copy its most significant bit to the parity bit b before setting it to 0. That will be y.
2. Let $u = y^2 - 1 \pmod q$ and $v = dy^2 + 1 \pmod q$. This means $x^2 = u/v \pmod q$.
3. Compute[18] $z = uv^3(uv^7)^{(q-5)/8} \pmod q$.
   (a) If $vz^2 = u \pmod q$ then $x' = z$.
   (b) If $vz^2 = -u \pmod q$ then calculate $x' = z * 2^{(q-1)/4} \pmod q$.
4. Using the parity bit b from the first step, if $b \neq$ the least significant bit of x' then $x = -x' \pmod q$, otherwise $x = x'$.
5. Return the decompressed point (x, y).

Implementations of Ed25519 (such as Monero) typically use the generator $G = (x, 4/5)$ [29], where x is the 'even', or $b = 0$, variant based on point decompression of $y = 4/5 \pmod q$.

2.4.3 EdDSA signature algorithm

Bernstein and his team have developed a number of basic algorithms based on curve Ed25519.[19] For illustration purposes we will describe a highly optimized and secure alternative to the ECDSA signature scheme which, according to the authors, allows producing over 100 000 signatures per second using a commodity Intel Xeon processor [29]. The algorithm can also be found described in Internet RFC8032 [40]. Note this is a very Schnorr-like signature scheme.

Among other things, instead of generating random integers every time, it uses a hash value derived from the private key of the signer and the message itself. This circumvents security flaws related to the implementation of random number generators. Also, another goal of the algorithm is to avoid accessing secret or unpredictable memory locations to prevent so-called *cache timing attacks* [29].

We provide here an outline of the steps performed by the algorithm for illustration purposes only. A complete description and sample implementation in the Python language can be found in [40].

[18] Since $q = 2^{255} - 19 \equiv 5 \pmod 8$, $(q-5)/8$ and $(q-1)/4$ are integers.

[19] See [30] for efficient group operations in Twisted Edwards EC (i.e. point addition, doubling, mixed addition, etc). See [26] for efficient modular arithmetic.

src/crypto/crypto_ops_builder/ref10CommentedCombined/

Signature

1. Let h_k be a hash $\mathcal{H}(k)$ of the signer's private key k. Compute α as a hash $\alpha = \mathcal{H}(h_k, \mathfrak{m})$ of the hashed private key and message. Depending on implementation, $\mathfrak{m}$ could be the actual message or its hash [40].
2. Calculate αG and the challenge $ch = \mathcal{H}([\alpha G], K, \mathfrak{m})$.
3. Calculate the response $r = \alpha + ch \cdot k$.
4. The signature is the pair $(\alpha G, r)$.

Verification

Verification is performed as follows

1. Compute $ch' = \mathcal{H}([\alpha G], K, \mathfrak{m})$.
2. If the equality $2^c rG = 2^c \alpha G + 2^c ch' * K$ holds then the signature is valid.

The 2^c term comes from Bernstein *et al.*'s general form of the EdDSA algorithm [30]. According to that paper, though it isn't required for adequate verification, removing 2^c provides stronger equations.

The public key K can be any EC point, but we only want to use points in the generator G's subgroup. Multiplying by the cofactor 2^c ensures all points are in that subgroup. Alternatively, verifiers could check $lK \overset{?}{=} 0$, which only works if K is in the subgroup. We do not know of any weaknesses caught by these precautions, though as we will see the later method is important in Monero.

In this signature scheme we store one EC point and one scalar, have one public EC key, and verify with:

src/ringct/rctOps.cpp

PA	Point addition for some points A, B: $A + B$ [1]
KBSM	Known-base scalar multiplications for some integer a: aG [1]
VBSM	Variable-base scalar multiplications for some integer a, and point P: aP [1]

Why it works

$$\begin{aligned} 2^c rG &= 2^c(\alpha + \mathcal{H}([\alpha G], K, \mathfrak{m}) \cdot k) \cdot G \\ &= 2^c \alpha G + 2^c \mathcal{H}([\alpha G], K, \mathfrak{m}) \cdot K \end{aligned}$$

Binary representation

By default, an EdDSA signature would need $64 + 32$ bytes for the EC point αG and scalar r. However, RFC8032 assumes point αG is compressed, which reduces space requirements to only $32 + 32$ bytes. We include the public key K, which implies $32 + 32 + 32$ total bytes.

CHAPTER 3

Ring Signatures

Ring signatures are composed of a ring and a signature. Each *signature* is generated with a single private key and a set of unrelated public keys. Each *ring* is the set of public keys comprising the private key's public key, and the set of unrelated public keys. Somebody verifying a signature would not be able to tell which ring member corresponds to the private key that created it.

Ring signatures were originally called *Group Signatures* because they were thought of as a way to prove a signer belongs to a group, without necessarily identifying him. In the context of Monero they will allow for unforgeable, signer-ambiguous transactions that leave currency flows largely untraceable.

Ring signature schemes display a number of properties that will be useful for producing confidential transactions:

Signer Ambiguity An observer should be able to determine the signer must be a member of the ring (except with negligible probability), but not which member.[1] Monero uses this to obfuscate the origin of funds in each transaction.

[1] Anonymity for an action is usually in terms of an 'anonymity set', which is 'all the people who could have possibly taken that action'. The largest anonymity set is 'humanity', and for Monero it is the so-called *mixin level* v plus the real signer. Mixin refers to how many fake members each ring signature has. If the mixin is $v = 4$ then there are 5 possible signers. Expanding anonymity sets makes it progressively harder to find real actors.

Linkability If a private key is used to sign two different messages then the messages will become linked.[2] As we will show, this property is used to prevent double-spending attacks in Monero (except with negligible probability).

Unforgeability No attacker can forge a signature except with negligible probability.[3] This is used to prevent theft of Monero funds by those not in possession of the appropriate private keys.

3.1 Linkable Spontaneous Anonymous Group (LSAG) signatures

Originally (Chaum in [32]), group signature schemes required the system be set up, and in some cases managed, by a trusted person in order to prevent illegitimate signatures, and, in a few schemes, adjudicate disputes. Relying on a *group secret* is not desirable since it creates a disclosure risk that could undermine anonymity. Moreover, requiring coordination between group members (i.e. for setup and management) is not scalable beyond small groups or within companies.

Liu *et al.* presented a more interesting scheme in [43] building on the work of Rivest *et al.* in [56]. The authors detailed a group signature algorithm characterized by three properties: *anonymity, linkability,* and *spontaneity.* In other words, the owner of a private key could produce one anonymous signature by selecting any set of co-signers from a list of candidate public keys, without needing to collaborate with anyone.[4]

Signature

Ring signature schemes are very Schnorr-like. In fact, our signature scheme in Section 2.3.5 can be considered a one-key ring signature. We get to two keys by, instead of defining r right away, generating a decoy r' and creating a new challenge to define r with.

Let $\mathfrak{m}$ be the message to sign, $\mathcal{R} = \{K_1, K_2, ..., K_n\}$ a set of distinct public keys (a group/ring), and k_π the signer's private key corresponding to his public key $K_\pi \in \mathcal{R}$, where π is a secret index. Assume the existence of two hash functions, $\mathcal{H}_n$ and $\mathcal{H}_p$, mapping to integers from 1 to l,[5] and curve points in EC,[6,7] respectively.

[2] The linkability property does not apply to non-signing public keys. That is, a ring member whose public key has been mixed into different signatures will not be linked.

[3] Certain ring signature schemes, including the one in Monero, are strong against adaptive chosen-message and adaptive chosen-public-key attacks. An attacker who can obtain legitimate signatures for messages and corresponding to specific public keys in rings of his choice cannot discover how to forge the signature of even one message. This is called *existential unforgeability*; see [61] and [43].

[4] In the LSAG scheme linkability only applies to signatures using rings with the same members and in the same order, the 'exact same ring'. It is really "one anonymous signature per ring". Linkable signatures can be attached to different messages.

[5] In Monero, the hash function $\mathcal{H}_n(x)$ = sc_reduce32($Keccak(x)$) where *Keccak* is the basis of SHA3 and sc_reduce32() puts the 256 bit result in the range 1 to l.

[6] It doesn't matter if points from $\mathcal{H}_p$ are compressed or not. They can always be decompressed.

[7] Monero uses a hash function that returns curve points directly, rather than computing some integer that is then multiplied by G. $\mathcal{H}_p$ would be broken if someone discovered a way to find n_x such that $n_xG = \mathcal{H}_p(x)$.

1. Compute key image $\tilde{K} = k_\pi \mathcal{H}_p(\mathcal{R})$.
2. Generate random number $\alpha \in_R \mathbb{Z}_l$ and fake responses $r_i \in_R \mathbb{Z}_l$ for $i \in \{1, 2, ..., n\}$ but excluding $i = \pi$.
3. Calculate
$$c_{\pi+1} = \mathcal{H}_n(\mathcal{R}, \tilde{K}, \mathfrak{m}, [\alpha G], [\alpha \mathcal{H}_p(\mathcal{R})])$$
4. For $i = \pi + 1, \pi + 2, ..., n, 1, 2, ..., \pi - 1$ calculate, replacing $n + 1 \rightarrow 1$,
$$c_{i+1} = \mathcal{H}_n(\mathcal{R}, \tilde{K}, \mathfrak{m}, [r_i G + c_i K_i], [r_i \mathcal{H}_p(R) + c_i \tilde{K}])$$
5. Define the real response r_π such that $\alpha = r_\pi + c_\pi k_\pi \pmod{l}$.

The ring signature contains the signature $\sigma(\mathfrak{m}) = (c_1, r_1, r_2, ..., r_n)$, the key image $\tilde{K}$, and the ring $\mathcal{R}$.

Verification

Verification means proving $\sigma(\mathfrak{m})$ is a valid signature created by a private key corresponding to a public key in $\mathcal{R}$, and is done in the following manner:

1. Check $l\tilde{K} \stackrel{?}{=} 0$.
2. For $i = 1, 2, ..., n$ iteratively compute, replacing $n + 1 \rightarrow 1$,
$$c'_{i+1} = \mathcal{H}_n(\mathcal{R}, \tilde{K}, \mathfrak{m}, [r_i G + c_i K_i], [r_i \mathcal{H}_p(\mathcal{R}) + c_i \tilde{K}])$$
3. If $c'_1 = c_1$ then the signature is valid. Note that c'_1 is the last term calculated.

We must check $l\tilde{K} \stackrel{?}{=} 0$ because it is possible to add an EC point from the subgroup of size h (the cofactor) to $\tilde{K}$ and, if all c_i are multiples of h (which we could achieve with automated trial and error using different α and r_i values), make h unlinked valid signatures using the same ring and signing key.[8] This is because an EC point multiplied by its subgroup's order is zero.[9]

src/cryptonote_core/cryptonote_core.cpp check_tx_inputs_keyimages_domain()

To be clear, given some point K in the subgroup of order l, some point K^h with order h, and an integer c divisible by h:
$$\begin{aligned} c * (K + K^h) &= cK + cK^h \\ &= cK + 0 \end{aligned}$$

In this scheme we store (1+n) integers, have one EC key image, use n public keys, and verify with:

src/ringct/rctOps.cpp

VBSM	Variable-base scalar multiplications for some integer a, and point P: aP	[1]
DVBA	Double-variable-base addition for some integers a, b, and point B: $aG + bB$	[n]
VBA	Variable-base additions for some integers a, b, and points A, B: $aA + bB$	[n]

[8] We are not concerned with points from other subgroups because the output of $\mathcal{H}_n$ is confined to $\mathbb{Z}_l$. For EC order $N = hl$, all divisors of N (and hence, possible subgroups) are either multiples of l (a prime) or divisors of h.

[9] In Monero's early history this was not checked for. Fortunately, it was not exploited before a fix was implemented in April, 2017 (v5 of the protocol) [55] .

Why it works

We can informally convince ourselves the algorithm works by going through an example. Consider ring $R = \{K_1, K_2, K_3\}$ with $k_\pi = k_2$. First the signature:

1. Create key image: $\tilde{K} = k_\pi \mathcal{H}_p(\mathcal{R})$
2. Generate random numbers: α, r_1, r_3
3. Seed the signature loop:
$$c_3 = \mathcal{H}_n(..., [\alpha G], [\alpha \mathcal{H}_p(\mathcal{R})])$$
4. Iterate:
$$c_1 = \mathcal{H}_n(..., [r_3 G + c_3 K_3], [r_3 \mathcal{H}_p(\mathcal{R}) + c_3 \tilde{K}])$$
$$c_2 = \mathcal{H}_n(..., [r_1 G + c_1 K_1], [r_1 \mathcal{H}_p(\mathcal{R}) + c_1 \tilde{K}])$$
5. Close the loop by responding: $r_2 = \alpha - c_2 k_2 \pmod l$

We can substitute α into c_3 to see where the word 'ring' comes from:

$$c_3 = \mathcal{H}_n(..., [(r_2 + c_2 k_2)G], [(r_2 + c_2 k_2)\mathcal{H}_p(\mathcal{R})])$$
$$c_3 = \mathcal{H}_n(..., [r_2 G + c_2 K_2 \ ], [r_2 \mathcal{H}_p(\mathcal{R}) + c_2 \tilde{K} \ \ ])$$

Then verification using $\mathcal{R}$, $\tilde{K}$, and $\sigma(\mathfrak{m}) = (c_1, r_1, r_2, r_3)$:

1. We use r_1 and c_1 to compute
$$c'_2 = \mathcal{H}_n(..., [r_1 G + c_1 K_1], [r_1 \mathcal{H}_p(\mathcal{R}) + c_1 \tilde{K}])$$
2. From when we made the signature, we see $c'_2 = c_2$. With r_2 and c'_2 we compute
$$c'_3 = \mathcal{H}_n(..., [r_2 G + c'_2 K_2], [r_2 \mathcal{H}_p(\mathcal{R}) + c'_2 \tilde{K}])$$
3. We can easily see that $c'_3 = c_3$ by substituting c_2 for c'_2. Using r_3 and c'_3 we get
$$c'_1 = \mathcal{H}_n(..., [r_3 G + c'_3 K_3], [r_3 \mathcal{H}_p(\mathcal{R}) + c'_3 \tilde{K}])$$

No surprises here: $c'_1 = c_1$ if we substitute c_3 for c'_3.

Linkability

Given a fixed set of public keys $\mathcal{R}$ and two valid signatures for different messages,

$$\sigma(\mathfrak{m}) = (c_1, r_1, ..., r_n) \text{ with } \tilde{K}, \text{ and}$$
$$\sigma'(\mathfrak{m}') = (c'_1, r'_1, ..., r'_n) \text{ with } \tilde{K}',$$

if $\tilde{K} = \tilde{K}'$ then clearly both signatures come from the same signing ring and private key.

While an observer could link σ and σ', he wouldn't know which K_i in $\mathcal{R}$ was the culprit without solving the DLP or auditing $\mathcal{R}$ in some way (such as learning all k_i with $i \neq \pi$, or learning k_π).[10]

[10] LSAG is unforgeable, meaning no attacker could make a valid ring signature without knowing a private key.

3.2 Back's Linkable Spontaneous Anonymous Group (bLSAG) signatures

In the LSAG signature scheme, linkability of signatures using the same private key can only be guaranteed if the ring is constant. This is obvious from the definition $\tilde{K} = k_\pi \mathcal{H}_p(\mathcal{R})$.

In this section we present an enhanced version of the LSAG algorithm where linkability is independent of the ring's co-signers.

The modification was unraveled in [61] based on a publication by A. Back [25] regarding the CryptoNote [64] ring signature algorithm (previously used in Monero, and now deprecated), which was inspired by Fujisaki and Suzuki's work in [35].

Signature

1. Calculate key image $\tilde{K} = k_\pi \mathcal{H}_p(K_\pi)$.
2. Generate random number $\alpha \in_R \mathbb{Z}_l$ and random numbers $r_i \in_R \mathbb{Z}_l$ for $i \in \{1, 2, ..., n\}$ but excluding $i = \pi$.
3. Compute
$$c_{\pi+1} = \mathcal{H}_n(\mathfrak{m}, [\alpha G], [\alpha \mathcal{H}_p(K_\pi)])$$
4. For $i = \pi + 1, \pi + 2, ..., n, 1, 2, ..., \pi - 1$ calculate, replacing $n + 1 \rightarrow 1$,
$$c_{i+1} = \mathcal{H}_n(\mathfrak{m}, [r_i G + c_i K_i], [r_i \mathcal{H}_p(K_i) + c_i \tilde{K}])$$
5. Define $r_\pi = \alpha - c_\pi k_\pi \pmod{l}$.

The signature will be $\sigma(\mathfrak{m}) = (c_1, r_1, ..., r_n)$, with key image $\tilde{K}$ and ring $\mathcal{R}$.

As in the original LSAG scheme, verification takes place by recalculating the value c_1.

Just like LSAG, in this scheme we store $(1+n)$ integers, have one EC key image, use n public keys, and verify with: src/ringct/rctOps.cpp

VBSM	Variable-base scalar multiplications for some integer a, and point P: aP	[1]
DVBA	Double-variable-base addition for some integers a, b, and point B: $aG + bB$	[n]
VBA	Variable-base additions for some integers a, b, and points A, B: $aA + bB$	[n]

Correctness can also be demonstrated (i.e. 'how it works') in a way similar to the LSAG scheme.

The alert reader will no doubt notice the key image $\tilde{K}$ only depends on the true signer's keys. In other words, two signatures will now be linkable if and only if the same private key was used to

If he invents a fake $\tilde{K}$ and seeds his signature computation with $c_{\pi+1}$, then, not knowing k_π, he can't calculate a number $r_\pi = \alpha - c_\pi k_\pi$ that would produce $[r_\pi G + c_\pi K_\pi] = \alpha G$. A verifier would reject his signature. Liu *et al.* prove forgeries that manage to pass verification are extremely improbable [43].

create them. In bLSAG, rings are just involved in obfuscating each signer's identity. bLSAG also simplifies the scheme by removing $\mathcal{R}$ and $\tilde{K}$ from the hash that calculates c_i.

This approach to linkability will prove to be more useful for Monero than the one offered by the LSAG algorithm, as it will allow detecting double-spending attempts without putting constraints on the ring members used.

3.3 Multilayer Linkable Spontaneous Anonymous Group (MLSAG) signatures

In order to sign multi-input transactions, one has to sign with m private keys. In [61], Shen Noether *et al.* describe a multi-layered generalization of the bLSAG signature scheme applicable when we have a set of $n \cdot m$ keys; that is, the set

$$\mathcal{R} = \{K_{i,j}\} \quad \text{for} \quad i \in \{1, 2, ..., n\} \quad \text{and} \quad j \in \{1, 2, ..., m\}$$

where we know the private keys $\{k_{\pi,j}\}$ corresponding to the subset $\{K_{\pi,j}\}$ for some index $i = \pi$.

Such an algorithm would address our multi-input needs if we generalize the notion of linkability.

Linkability: if any private key $k_{\pi,j}$ is used in 2 different signatures, then these signatures will be automatically linked.

Signature

1. Calculate key images $\tilde{K}_j = k_{\pi,j}\mathcal{H}_p(K_{\pi,j})$ for all $j \in \{1, 2, ..., m\}$.
2. Generate random numbers $\alpha_j \in_R \mathbb{Z}_l$, and $r_{i,j} \in_R \mathbb{Z}_l$ for $i \in \{1, 2, ..., n\}$ (except $i = \pi$) and $j \in \{1, 2, ..., m\}$.
3. Compute[11]
$$c_{\pi+1} = \mathcal{H}_n(\mathfrak{m}, [\alpha_1 G], [\alpha_1 \mathcal{H}_p(K_{\pi,1})], ..., [\alpha_m G], [\alpha_m \mathcal{H}_p(K_{\pi,m})])$$
4. For $i = \pi+1, \pi+2, ..., n, 1, 2, ..., \pi-1$ calculate, replacing $n+1 \rightarrow 1$,
$$c_{i+1} = \mathcal{H}_n(\mathfrak{m}, [r_{i,1}G + c_i K_{i,1}], [r_{i,1}\mathcal{H}_p(K_{i,1}) + c_i \tilde{K}_1], ..., [r_{i,m}G + c_i K_{i,m}], [r_{i,m}\mathcal{H}_p(K_{i,m}) + c_i \tilde{K}_m])$$
5. Define $r_{\pi,j} = \alpha_j - c_\pi k_{\pi,j} \pmod{l}$.

The signature will be $\sigma(\mathfrak{m}) = (c_1, r_{1,1}, ..., r_{1,m}, ..., r_{n,1}, ..., r_{n,m})$, with key images $(\tilde{K}_1, ..., \tilde{K}_m)$.

[11] Monero MLSAG uses so-called 'key prefixing', which, when implemented, was thought to allow better security proofs for multi-user Schnorr-like signatures. Recent research suggests it isn't necessary. [41] Each challenge contains an explicit public key like this:

$$c_{\pi+1} = \mathcal{H}_n(\mathfrak{m}, K_{\pi,1}, [\alpha_1 G], [\alpha_1 \mathcal{H}_p(K_{\pi,1})], ..., K_{\pi,m}, [\alpha_m G], [\alpha_m \mathcal{H}_p(K_{\pi,m})])$$

src/ringct/
rctSigs.cpp
MLSAG_gen()

Verification

The verification of a signature is done in the following manner:

1. For all $j \in \{1, ..., m\}$ check $l\tilde{K}_j \stackrel{?}{=} 0$.
2. For $i = 1, ..., n$ compute, replacing $n + 1 \to 1$,
   $c'_{i+1} = \mathcal{H}_n(\mathfrak{m}, [r_{i,1}G + c_iK_{i,1}], [r_{i,1}\mathcal{H}_p(K_{i,1}) + c_i\tilde{K}_1], ..., [r_{i,m}G + c_iK_{i,m}], [r_{i,m}\mathcal{H}_p(K_{i,m}) + c_i\tilde{K}_m])$
3. If $c'_1 = c_1$ then the signature is valid.

Why it works

Just as with the original LSAG algorithm, we can readily observe that

- If $i \neq \pi$, then clearly the values c'_{i+1} are calculated as described in the signature algorithm.
- If $i = \pi$ then, since $r_{\pi,j} = \alpha_j - c_\pi k_{\pi,j}$ closes the loop,

  $$r_{\pi,j}G + c_\pi K_{\pi,j} = (\alpha_j - c_\pi k_{\pi,j})G + c_\pi K_{\pi,j} = \alpha_j G$$

  and

  $$r_{\pi,j}\mathcal{H}_p(K_{\pi,j}) + c_\pi\tilde{K}_j = (\alpha_j - c_\pi k_{\pi,j})\mathcal{H}_p(K_{\pi,j}) + c_\pi\tilde{K}_j = \alpha_j\mathcal{H}_p(K_{\pi,j})$$

  In other words, it holds also that $c'_{\pi+1} = c_{\pi+1}$.

Linkability

If a private key $k_{\pi,j}$ is re-used to make any signature, the corresponding key image $\tilde{K}_j$ supplied in the signature will reveal it. This observation matches our generalized definition of linkability.[12]

Space and verification requirements

In this scheme we store $(1+m * n)$ integers, have m EC key images, use $m * n$ public keys, and verify with:

src/ringct/rctOps.cpp

VBSM	Variable-base scalar multiplications for some integer a, and point P: aP	[m]
DVBA	Double-variable-base addition for some integers a, b, and point B: $aG + bB$	$[m * n]$
VBA	Variable-base additions for some integers a, b, and points A, B: $aA + bB$	$[m * n]$

[12] As with LSAG, linked MLSAG signatures do not indicate which public key was used to sign it. However, if the key image's sub-loops' rings have only one key in common, the culprit is obvious.

3.4 Borromean ring signatures

We will see in later sections of this report it is necessary to prove transaction amounts are within an expected range. This can be accomplished with ring signatures. However, linkable signatures here would allow observers to link outputs using common amounts, or guess and check the entire transaction history. Fortunately, excluding linkability allows us to select more efficient algorithms in terms of space consumed and verification speed.

In this context, and for the *singular purpose* of proving amount ranges, Monero uses a signature scheme developed by G. Maxwell, which he described in [46].[13] We present here a complete version of the scheme for educational purposes.

Assume we have a set $\mathcal{R}$ of public keys $\{K_{i,j_i}\}$ for $i \in \{1, 2, ..., n\}$ and $j_i \in \{1, 2, ..., m_i\}$. In other words, $\{K_{i,j_i}\}$ is like a bookshelf of public keys with n shelves, and on each i^{th} shelf are m_i public keys ordered from 1 to m_i. The amount of keys m_i can be different for each shelf i, hence the subscript.[14]

Furthermore, assume for each i there is an index π_i (which can be different for each i) such that the signer knows private key k_{i,π_i} corresponding to K_{i,π_i}. Following the analogy, a signer knows one private key for the $j_i = {\pi_i}^{\text{th}}$ public key on each shelf i in the bookshelf.

In what follows, $\mathfrak{m}$ is a hash of the message to be signed with keys $\{K_{i,j}\}$.

Signature

1. For each shelf $i \in \{1, ..., n\}$ [skip this entire step for any i where $\pi_i = m_i$]:
   (a) generate a random value $\alpha_i \in_R \mathbb{Z}_l$,
   (b) seed the shelf's loop: set $c_{i,\pi_i+1} = \mathcal{H}_n(\mathfrak{m}, [\alpha_i G])$,
   (c) build first half of loop from seed: if $\pi_i + 1 = m_i$ skip this step, otherwise for $j_i = \pi_i + 1, ..., m_i - 1$ generate random numbers $r_{i,j_i} \in_R \mathbb{Z}_l$ and compute
   $$c_{i,j_i+1} = \mathcal{H}_n(\mathfrak{m}, [r_{i,j_i} G + c_{i,j_i} K_{i,j_i}])$$
2. For $i \in \{1, ..., n\}$ generate random numbers $r_{i,m_i} \in_R \mathbb{Z}_l$. Instead of making separate challenges with the last key on each shelf, take all r_{i,m_i}, c_{i,m_i}, and K_{i,m_i}, and combine them in the connector
   $$c_1 = \mathcal{H}_n(\mathfrak{m}, [r_{1,m_1} G + c_{1,m_1} K_{1,m_1}], ..., [r_{n,m_n} G + c_{n,m_n} K_{n,m_n}])$$
   Note: if any $\pi_i = m_i$, generate α_i instead of r_{i,m_i} and put $\alpha_i G$ in c_1.

[13] Monero's first iteration of 'range proofs' used Aggregate Schnorr Non-Linkable Ring Signatures (ASNL) [61]. According to the author of ASNL it reduces to a kind of Borromean ring signature [21], but since the latter is more general and secure it was chosen for final implementation in November, 2016.

[14] In Monero, range proofs require shelves with exactly 2 keys corresponding to each digit of an amount represented in binary. This means all $m_i = 2$, so the Borromean scheme can be implemented in a simpler form (see Chapter 4).

3. For each shelf $i \in \{1, .., n\}$:

   (a) build second half of loop from connector: if $\pi_i = 1$ skip this, otherwise for $j_i = 1, ..., \pi_i - 1$ generate random numbers $r_{i,j_i} \in_R \mathbb{Z}_l$ and compute [we interpret references to $c_{i,1}$ as c_1]
   $$c_{i,j_i+1} = \mathcal{H}_n(\mathfrak{m}, [r_{i,j_i}G + c_{i,j_i}K_{i,j_i}])$$
   (b) tie loop ends together: set r_{i,π_i} such that $\alpha_i = r_{i,\pi_i} + c_{i,\pi_i}k_{i,\pi_i}$.

The signature is
$$\sigma = (c_1, r_{1,1}, ..., r_{1,m_1}, r_{2,1} ..., r_{2,m_2}, ..., r_{n,m_n})$$
Using the connector lets us tie many subloops together, instead of a separate ring signature for each. This means $n-1$ fewer scalars to store these signatures. Note this signature scheme reduces to our Schnorr-like signature from Section 2.3.4 when $\mathcal{R}$ only contains one key.

Verification

Given $\mathfrak{m}$, $\mathcal{R}$, and σ, verification is performed as follows:

1. For $i \in \{1, ..., n\}$ and $j_i = 1, ..., m_i$ build each loop:
   $$L'_{i,j_i} = r_{i,j_i}G + c'_{i,j_i}K_{i,j_i}$$
   $$c'_{i,j_i+1} = \mathcal{H}_n(\mathfrak{m}, L'_{i,j_i})$$
   Interpret any $c'_{i,1}$ as c_1. Also, it is not necessary to compute c'_{i,m_i+1}.
2. Compute the connector $c'_1 = \mathcal{H}_n(\mathfrak{m}, L'_{1,m_1}, ..., L'_{n,m_n})$.

The signature is valid if $c'_1 = c_1$.

In this scheme we store $(1+\sum_{i=1}^{n} m_i)$ integers, use $\sum_{i=1}^{n} m_i$ public keys, and verify with: src/ringct/rctOps.cpp

DVBA Double-variable-base addition for some integers a, b, and point B: $aG + bB$ $[\sum_{i=1}^{n} m_i]$

Why it works

1. For $j_i \neq \pi_i$ and for all i we can readily see that $c'_{i,j_i+1} = c_{i,j_i+1}$.
2. When $j_i = \pi_i$, for all i:
   $$\begin{aligned} L'_{i,\pi_i} &= r_{i,\pi_i}G + c'_{i,\pi_i}K_{i,\pi_i} \\ &= (\alpha_i - c_{i,\pi_i}k_{i,\pi_i})G + c'_{i,\pi_i}K_{i,\pi_i} \\ &= \alpha_i G - c_{i,\pi_i}k_{i,\pi_i}G + c'_{i,\pi_i}k_{i,\pi_i}G \\ &= \alpha_i G \end{aligned}$$
   In other words, $c'_{i,\pi_i+1} = \mathcal{H}_n(\mathfrak{m}, [\alpha_i G]) = c_{i,\pi_i+1}$.

Therefore we can conclude the verification step identifies valid signatures (except in the case of a forgery, which can only occur with negligible probability).

CHAPTER 4

Pedersen Commitments

Generally speaking, a cryptographic *commitment scheme* is a way of committing to a value without revealing the value itself.

For example, in a coin-flipping game Alice could privately commit to one outcome (i.e. 'call it') before Bob flips the coin by publishing her committed value hashed with secret data. After Bob flips the coin, Alice could declare which value she committed to and prove it by publishing her secret data. Bob could then verify her claim.

In other words, assume that Alice has a secret string *blah* and the value she wants to commit to is *heads*. She could simply hash $h = \mathcal{H}(blah, heads)$ and give h to Bob. Bob flips a coin, Alice tells Bob about *blah* and informs him she committed to *heads*, and then Bob calculates $h' = \mathcal{H}(blah, heads)$. If $h' = h$, then he knows Alice called *heads* before the coin flip.

Alice uses the so-called 'salt' *blah* so Bob can't just guess $\mathcal{H}(heads)$ and $\mathcal{H}(tails)$ before his coin flip, and figure out she committed to *heads*.

4.1 Pedersen commitments

A *Pedersen commitment* [54] is a commitment that has the property of being *additively homomorphic*. If $C(a)$ and $C(b)$ denote the commitments for values a and b respectively, then $C(a + b) = C(a) + C(b)$.[1] This property is useful when committing transaction amounts, as one

[1] Additively homomorphic in this context means addition is preserved when you transform scalars into EC points by applying, for scalar x, $x \to xG$.

could prove, for instance, that inputs equal outputs, without revealing the amounts at hand.

Fortunately, Pedersen commitments are easy to implement with elliptic curve cryptography, as the following holds trivially

$$aG + bG = (a + b)G$$

Clearly, by defining a commitment as simply $C(a) = aG$, we could easily create cheat tables of commitments to help us recognize common amounts a.

To attain information-theoretic privacy, one needs to add a secret *blinding factor* and another generator H, such that it is unknown for which value of γ the following holds: $H = \gamma G$. The hardness of the discrete logarithm problem ensures calculating γ from H is infeasible. In the case of Monero, $H = \mathcal{H}_p(G)$.[2]

We can then define the commitment to an amount a as $C(x, a) = xG + aH$, where x is a blinding factor that prevents observers from guessing a (for example: if you just commit $C(a = 1)$, it is trivial to guess and check).

Commitment $C(x, a)$ is information-theoretically private because there are many possible combinations of x and a that would output the same C.[3] If x is truly random, an attacker would have literally no way to figure out a [45, 59].

4.2 Amount commitments

Owning cryptocurrency is not like a bank account, where a person's balance exists as a single value in a database. Rather, a person owns a bunch of transaction *outputs*. Each output has an 'amount', and the sum of amounts in all *unspent* outputs owned is considered a person's balance.

To send cryptocurrency to someone else, we create a transaction. A transaction references old outputs as *inputs* and addresses new outputs to recipients. Once an output has been referenced, i.e. 'spent', it can never be referenced/spent again.

Since it is rare for input amounts to equal intended output amounts, most transactions include 'change', an output that sends excess back to the sender. We will elaborate on these topics in Chapter 5.

In Monero, transaction amounts are hidden using a technique called RingCT, first implemented in January, 2017 (v4 of the protocol). While transaction verifiers don't know how much Monero

[2] The Monero codebase has a function *to_point*() that maps scalars to EC points. For commitments, $H = to\_point(Keccak(G))$.

src/ringct/rctTypes.h

[3] Basically, there are many x' and a' such that $x' + a'\gamma = x + a\gamma$. A committer knows one combination, but an attacker has no way to know which one. Furthermore, even the committer can't find another combination without solving the DLP for γ.

is contained in each input and output, they still need to prove the sum of input amounts equals the sum of output amounts.

In other words, if we had a transaction with inputs containing amounts $a_1, ..., a_m$ and outputs with amounts $b_1, ..., b_p$, then an observer would justifiably expect that:

$$\sum_j a_j - \sum_t b_t = 0$$

Since commitments are additive and we don't know γ, we could easily prove our inputs equal outputs to observers by making the sum of commitments to input and output amounts equal zero:[4]

$$\sum_j C_{j,in} - \sum_t C_{t,out} = 0$$

To avoid sender identifiability, Shen Noether proposed [61] verifying that commitments sum to a certain non-zero value:

$$\begin{aligned}\sum_j C_{j,in} - \sum_t C_{t,out} &= zG \\ \sum_j (x_j G + a_j H) - \sum_t (y_t G + b_t H) &= zG \\ \sum_j x_j - \sum_t y_t &= z\end{aligned}$$

The reasons why this is useful will become clear in Chapter 5 when we discuss the structure of transactions.

Note that $C = zG$ is called a *commitment to zero* because we can make a signature with z, which proves that there is no H component to C (assuming γ is unknown). In other words $C = zG + 0H$.

4.3 Range proofs

One problem with additive commitments is that, if we have commitments $C(a_1)$, $C(a_2)$, $C(b_1)$, and $C(b_2)$ and we intend to use them to prove that $(a_1 + a_2) - (b_1 + b_2) = 0$, then those commitments would still apply if one value in the equation were 'negative'.

For instance, we could have $a_1 = 6$, $a_2 = 5$, $b_1 = 21$, and $b_2 = -10$.

$$(6 + 5) - (21 + -10) = 0$$

where

$$21G + -10G = 21G + (l - 10)G = (l + 11)G = 11G$$

Since $-10 = l - 10$, we have effectively created l more Moneroj (over 7.2x10^{74}!) than we put in.

[4] Recall from Section 2.3.1 we can subtract a point by inverting its coordinates. If $P = (x, y)$, $-P = (-x, y)$. Recall also that negations of field elements are calculated $\pmod q$, so $(-y \pmod q)$.

The solution addressing this issue in Monero is to prove each output amount is in a certain range using the Borromean signature scheme described in Section 3.4. This method treats each bit in the binary representation of the amount as a separate amount which we can commit to zero.

Given a commitment $C(b)$ with blinding factor y_b for amount b, use the binary representation $(b_0, b_1, ..., b_{k-1})$ such that

$$b = b_0 2^0 + b_1 2^1 + ... + b_{k-1} 2^{k-1}$$

Generate random numbers $y_0, ..., y_{k-1} \in_R \mathbb{Z}_l$ to be used as blinding factors. Define also Pedersen commitments for each b_i, $C_i = y_i G + b_i 2^i H$, and derive public keys $\{C_i, C_i - 2^i H\}$.

Clearly one of those public keys will equal $y_i G$:

$$\begin{aligned} &\text{if } b_i = 0 \text{ then} \quad C_i &&= y_i G + 0H &&= y_i G \\ &\text{if } b_i = 1 \text{ then} \quad C_i - 2^i H &&= y_i G + 2^i H - 2^i H &&= y_i G \end{aligned}$$

In other words, a blinding factor y_i will always be the private key corresponding to one of the points $\{C_i, C_i - 2^i H\}$. The point that equals $y_i G$ is a *commitment to zero*, because either $b_i 2^i = 0$ or $b_i 2^i - 2^i = 0$. We can prove a transaction output's amount b is in the range $[0, ..., 2^k - 1]$ by signing it using the Borromean Ring Signature scheme of Section 3.4 with the ring of public keys: src/ringct/rctSigs.cpp genBorromean()

$$\{\{C_0, C_0 - 2^0 H\}, ..., \{C_{k-1}, C_{k-1} - 2^{k-1} H\}\}$$

where we know the private keys $\{y_0, ..., y_{k-1}\}$ corresponding to each pair.

Resulting in a signature $\sigma = (c_1, r_{0,1}, r_{0,2}, r_{1,1} ..., r_{k-1,2})$

4.4 Range proofs in a blockchain

In the context of Monero we will use range proofs to prove there are valid amounts in the outputs of each transaction.

Transaction verifiers will have to check that the sum of each output's range proof commitments C_i equals its amount commitment C_b. For this to work we need to modify our definition of the blinding factor y_b: set $y_0, ..., y_{k-1} \in_R \mathbb{Z}_l$ and define $y_b = \sum_{i=0}^{k-1} y_i$. The following equation now holds src/ringct/rctSigs.cpp proveRange()

$$\sum_{i=0}^{k-1} C_i = C_b$$

We will store only the range proof commitments/keys C_i, the output amount's commitment C_b, and the signature σ's terms in the blockchain. The mining community can easily calculate $C_i - 2^i H$ and verify the Borromean ring signature for each output.

It will not be necessary for the receiver nor any other party to know the blinding factors y_i, as their sole purpose is proving a new output's amount is in range.

Since the Borromean signature scheme requires knowledge of y_i to produce a signature, any third party who verifies one can convince himself that each loop contains a commitment to zero, so total amounts must fall within range and money is not being artificially created.

In this scheme we store $(1{+}2*k)$ integers, use k public keys (commitments), and verify with:

DVBA Double-variable-base addition for some integers a, b, and point B: $aG + bB$ $[2*k]$

Verifiers need to calculate $C_i - 2^i H$ and $\sum_i C_i$, so verification times include:

PA Point addition for some points A, B: $A + B$ $[k-1]$
PS Point subtraction for some points A, B : $A - B$ $[k]$
KBSM Known-base scalar multiplications for some integer a: aG $[k]$

Monero uses a special function to compute $C_i - 2^i H$ and $\sum_i C_i = C$ that is supposed to be faster: src/ringct/rctSigs.cpp verRange()

VRSF Verify range proof special function $[k]$

For a total verification requirement: src/ringct/rctOps.cpp

VRSF Verify range proof special function $[k]$
DVBA Double-variable-base addition for some integers a, b, and point B: $aG + bB$ $[2*k]$

CHAPTER 5

Monero Transactions

5.1 User keys

Unlike Bitcoin, Monero users have two sets of private/public keys, (k^v, K^v) and (k^s, K^s), generated as described in Section 2.3.2.

The *address* of a user is the pair of public keys (K^v, K^s). Her private keys will be the corresponding pair (k^v, k^s).

Using two sets of keys allows function segregation. The rationale will become clear later in this chapter, but for the moment let us call private key k^v the *view key*, and k^s the *spend key*. A person can use their view key to determine if an output is addressed to them, and their spend key will allow them to send that output in a transaction (and retroactively figure out it has been spent).[1]

5.2 One-time addresses

To verify a transfer of money, observers need to know the money starts off being owned by the spender. How do they own it in the first place? Every Monero user has a public address which

[1] It is currently most common for the view key k^v to equal $\mathcal{H}_n(k^s)$. This means a person only needs to save their spend key k^s in order to access (view and spend) all of the outputs they own (spent and unspent). The spend key is typically represented as a series of 25 words (where the 25th word is a checksum). Other, less popular methods include: generating k^v and k^s as separate random numbers, or generating a random 12-word mnemonic a, where $k^s = sc\_reduce32(Keccak(a))$ and $k^v = sc\_reduce32(Keccak(Keccak(a)))$. [3]

src/wallet/
api/wallet.cpp
create()
wallet2.cpp
generate()
get_seed()

they may distribute to other users, who can then send money to that address via transaction outputs.

The public address is never used directly. Instead, a Diffie-Hellman-like exchange is applied to it, creating a unique *one-time address* for each transaction output to be paid to the user. In this way, even external observers who know all users' public addresses will not be able to identify which user received any given transaction output.

A recipient can spend their received outputs by signing a message with the one-time addresses, thereby proving they know the private keys and must therefore own what they are spending. We will gradually flesh out this concept.

Let's start with a very simple transaction, containing exactly one input and one output — a payment from Alice to Bob.

Bob has private/public keys (k^v_B, k^s_B) and (K^v_B, K^s_B), and Alice knows his public keys. A transaction could proceed as follows [64]:

1. Alice generates a random number $r \in_R \mathbb{Z}_l$, and calculates the one-time public key[2]
$$K^o = \mathcal{H}_n(rK^v_B)G + K^s_B$$
2. Alice sets K^o as the addressee of the payment, adds the value rG to the transaction data, and submits it to the network.

src/crypto/crypto.cpp derive_public_key()

3. Bob receives the data and sees the values rG and K^o. He can calculate $k^v_B rG = rK^v_B$. He can then calculate $K'^s_B = K^o - \mathcal{H}_n(rK^v_B)G$. When he sees that $K'^s_B = K^s_B$, he knows the transaction is addressed to him.[3]

   The private key k^v_B is called the *view key* because anyone who has it (and Bob's public spend key K^s_B) can calculate K'^s_B for every transaction output in the blockchain (record of transactions), and 'view' which ones are addressed to Bob.

src/crypto/crypto.cpp derive_subaddress_public_key()

4. The one-time keys for the output are:
$$K^o = \mathcal{H}_n(rK^v_B)G + K^s_B = (\mathcal{H}_n(rK^v_B) + k^s_B)G$$
$$k^o = \mathcal{H}_n(rK^v_B) + k^s_B$$

While Alice can calculate the public key K^o for the one-time address, she can not compute the corresponding private key k^o, since it would require either knowing Bob's spend key k^s_B, or solving the discrete logarithm problem for $K^s_B = k^s_B G$, which we assume to be hard. As will become clear

[2] In Monero, every instance of rk^vG is multiplied by the cofactor 8, so in this case Alice computes $8 * rK^v_B$ and Bob computes $8 * k^v_B rG$. As far as we can tell this serves no purpose (but *is* a convention that users should follow). Multiplying by cofactor ensures the resulting point is in G's subgroup, but if R and K^v don't share a subgroup to begin with, then the discrete logs r and k^v can't be used to make a shared secret regardless.

src/crypto/crypto.cpp generate_key_derivation()

[3] K'^s_B is computed with derive_subaddress_public_key() because normal address spend keys are stored at index 0 in the spend key lookup table, while subaddresses are at indices 1,2... This will make sense soon: see Section 5.3.

later in this chapter, without k^o Alice can't compute the output's key image, so she can never know for sure if Bob spends the output she sent him.[4]

A third party with Bob's *view key* can verify an output is addressed to Bob, yet without knowledge of the *spend key* this third party would not be able to spend that output nor know when it has been spent. Such a third party could be a trusted custodian, an auditor, a tax authority, etc. Somebody who could be allowed read access to the user's transaction history, without any further rights. This third party would also be able to decrypt the output's amount (to be explained in Section 5.6.1).

5.2.1 Multi-output transactions

Most transactions will contain more than one output. If nothing else, to transfer 'change' back to the sender.

Monero senders generate only one random value r. The value rG is normally known as the *transaction public key* and is published in the blockchain.

To ensure that all output addresses in a transaction with p outputs are different even in cases where the same addressee is used twice, Monero uses an output index. Every output from a transaction has an index $t \in \{1, ..., p\}$. By appending this value to the shared secret before hashing it, one can ensure the resulting one-time addresses are unique:

src/crypto/crypto.cpp derive_public_key()

$$\begin{aligned} K_t^o &= \mathcal{H}_n(rK_t^v, t)G + K_t^s = (\mathcal{H}_n(rK_t^v, t) + k_t^s)G \\ k_t^o &= \mathcal{H}_n(rK_t^v, t) + k_t^s \end{aligned}$$

5.3 Subaddresses

Monero users can generate subaddresses from each address [51]. Funds sent to a subaddress can be viewed and spent using its main address' view and spend keys. By analogy: an online bank account may have multiple balances corresponding to credit cards and deposits, yet they are all accessible and spendable from the same point of view – the account holder.

Subaddresses are convenient for receiving funds to the same place when a user doesn't want to link his activities together by publishing/using the same address. As we will see, an observer would

[4] Imagine Alice produces two transactions, each containing the same one-time output address K^o that Bob can spend. Since K^o only depends on r and K_B^v, there is no reason she can't do it. Bob can only spend one of those outputs because each one-time address only has one key image, so if he isn't careful Alice might trick him. She could make transaction 1 with a lot of money for Bob, and later transaction 2 with a small amount for Bob. If he spends the money in 2, he can never spend the money in 1. In fact, no one could spend the money in 1, effectively 'burning' it. Monero wallets have been designed to ignore the smaller amount in this scenario.

have to solve the DLP in order to determine a given subaddress is derived from any particular address [51].[5]

They are also useful for differentiating between received outputs. For example, if Alice wants to buy an apple from Bob on a Tuesday, Bob could write a receipt describing the purchase and make a subaddress for that receipt, then ask Alice to use that subaddress when she sends him the money. This way Bob can associate the money he receives with the apple he sold. We explore another way to distinguish between received outputs in the next section.

Bob generates his i^{th} subaddress ($i = 1, 2, ...$) from his address as a pair of public keys $(K^{v,i}, K^{s,i})$:

src/device/device_default.cpp get_subaddress_secret_key()

$$K^{s,i} = K^s + \mathcal{H}_n(k^v, i)G$$
$$K^{v,i} = k^v K^{s,i}$$

So,

$$K^{v,i} = k^v(k^s + \mathcal{H}_n(k^v, i))G$$
$$K^{s,i} = \quad (k^s + \mathcal{H}_n(k^v, i))G$$

5.3.1 Sending to a subaddress

Let's say Alice is going to send Bob money via his subaddress $(K_B^{v,1}, K_B^{s,1})$ with a simple one-input, one-output transaction.

1. Alice generates a random number $r \in_R \mathbb{Z}_l$, and calculates the one-time public key
$$K^o = \mathcal{H}_n(rK_B^{v,1})G + K_B^{s,1}$$

2. Alice sets K^o as the addressee of the payment, **adds the value** $rK_B^{s,1}$ **to the transaction data**, and submits it to the network.

3. Bob receives the data and sees the values $rK_B^{s,1}$ and K^o. **He can calculate** $k_B^v rK_B^{s,1} = rK_B^{v,1}$. He can then calculate $K_B'^{s,1} = K^o - \mathcal{H}_n(rK_B^{v,1})G$. When he sees that $K_B'^{s,1} = K_B^{s,1}$, he knows the transaction is addressed to him.

   src/crypto/crypto.cpp derive_subaddress_public_key()

   Bob only needs his private view key k_B^v and subaddress public spend key $K_B^{s,1}$ to find transaction outputs sent to his subaddress.

4. The one-time keys for the output are:
$$K^o = \mathcal{H}_n(rK_B^{v,1})G + K_B^{s,1} = (\mathcal{H}_n(rK_B^{v,1}) + k_B^{s,1})G$$
$$k^o = \mathcal{H}_n(rK_B^{v,1}) + k_B^{s,1}$$

[5] Prior to subaddresses, Monero users could simply generate many normal addresses. To view each addresses' balance, you needed to do a separate scan of the blockchain record. This was very inefficient. With subaddresses, users maintain a look-up table of (hashed) spend keys, so one scan of the blockchain takes the same amount of time for 1 subaddress, or 10,000 subaddresses.

Now, Alice' transaction public key is particular to Bob ($rK_B^{s,1}$ instead of rG). If she creates a p-output transaction with at least one output intended for a subaddress, Alice needs to make a unique transaction public key for each output $t \in 1, ..., p$. In other words, if Alice is sending to Bob's subaddress ($K_B^{v,1}, K_B^{s,1}$) and Carol's address (K_C^v, K_C^s), she will put two transaction public keys $\{r_1 K_B^{s,1}, r_2 G\}$ in the transaction data.[6]

src/cryptonote_core/cryptonote_tx_utils.cpp construct_tx_with_tx_key()

5.4 Integrated addresses

In order to differentiate between the outputs they receive, a recipient can request senders include a *payment ID* in transaction data.[7] For example, if Alice wants to buy an apple from Bob on a Tuesday, Bob could write a receipt describing the purchase and ask Alice to include the receipt's ID number when she sends him the money. This way Bob can associate the money he receives with the apple he sold.

Senders can communicate payment IDs in clear text, but manually including the IDs in transactions is inconvenient, and a privacy hazard for recipients, who might inadvertently expose their activities. In Monero, recipients can integrate payment IDs into their addresses, and provide those *integrated addresses*, containing (K^v, K^s, payment ID), to senders. Payment IDs can technically be integrated into any kind of address, including normal addresses, subaddresses, and multisignature addresses.[8]

src/common/base58.cpp encode_addr()

Senders addressing outputs to integrated addresses can encode payment IDs using the shared secret rK_t^v and a XOR operation, which recipients can then decode with the appropriate transaction public key and another XOR procedure [6]. Encoding payment IDs in this way allows senders to prove they made particular transaction outputs (i.e. for audits, refunds, etc.).

Binary operator XOR

The binary operator XOR evaluates two arguments and returns true if one, but not both, of the arguments is true [19]. Here is its truth table:

A	B	A XOR B
T	T	F
T	F	T
F	T	T
F	F	F

[6] In Monero subaddresses are prefixed with an '8', separating them from addresses, which are prefixed with '4'. This helps senders choose the correct procedure when constructing transactions.

[7] Currently, Monero only supports one payment ID per transaction.

[8] Since an observer can recognize the difference between transactions with and without payment IDs of any kind, using them is thought to make the Monero transaction history less uniform. Subaddresses ('disposable addresses' more generally) can be used for the same purpose, so payment IDs may be superfluous. Based on these ideas, there is some effort in the Monero development community to deprecate all forms of payment ID [4]. However, cryptocurrency exchanges find them useful and are slow to change, so payment IDs might remain a feature indefinitely. Also note integrated addresses have only ever been implemented for normal addresses.

In the context of computer science, XOR is equivalent to bit addition modulo 2. For example, the XOR of two bit pairs:

$$\begin{aligned} \text{XOR}(\{1,1\},\{1,0\}) &= \{1+1, 1+0\} \pmod 2 \\ &= \{0,1\} \end{aligned}$$

Examining the previous example, each of these produce the same output: XOR($\{1,1\},\{1,0\}$), XOR($\{0,0\},\{0,1\}$), XOR($\{1,0\},\{1,1\}$), or XOR($\{0,1\},\{0,0\}$). There are 2^{b} combinations of XOR inputs (with b bits each) for the same output, so if input $A \in_R \{1, ..., 2^{\text{b}}\}$, an observer who learned $C = \text{XOR}(A, B)$ could not gain any information about B.

At the same time, anyone who knows two of the elements A, B, C, where $C = \text{XOR}(A, B)$, can calculate the third element, such as $A = \text{XOR}(B, C)$. XOR indicates if two elements are different or the same, so knowing C and B is enough to expose A. A careful examination of the truth table reveals this to be true.

Encoding

The sender encodes the payment ID for inclusion in transaction data[9]

src/device/device_default.cpp encrypt_payment_id()

$$\begin{aligned} k_{\text{mask}} &= \mathcal{H}_n(rK_t^v, \text{pid\_tag}) \\ k_{\text{payment ID}} &= k_{\text{mask}} \rightarrow \text{reduced to bit length of payment ID} \\ \text{encoded payment ID} &= \text{XOR}(k_{\text{payment ID}}, \text{payment ID}) \end{aligned}$$

The output of a cryptographic hash function $\mathcal{H}$ is uniformly distributed across the range of possible outputs. In other words, for some input A, $\mathcal{H}(A) \in_R^D \mathbb{S}_H$ where $\mathbb{S}_H$ is the set of possible outputs from $\mathcal{H}$. We use $\in_R^D$ to indicate the function is deterministically random. $\mathcal{H}(A)$ produces the same thing every time, but its output is equivalent to a random number.

We include pid_tag to ensure k_{mask} is different from the component $\mathcal{H}_n(rK_t^v, t)$ in one-time output addresses.[10]

Decoding

Whichever recipient t the payment ID was created for can find it using his view key and the transaction public key rG:[11]

src/device/device.hpp decrypt_payment_id()

[9] In Monero payment IDs for integrated addresses are conventionally 64 bits long, while independent, clear text payment IDs are usually 256 bits long.

[10] In Monero, pid_tag = ENCRYPTED_PAYMENT_ID_TAIL = 141. In, for example, multi-input transactions we compute $\mathcal{H}_n(rK_t^v, t) \pmod l$ to ensure we are using a scalar less than the EC subgroup order, but since $\pmod l$ is 253 bits and payment IDs are only 64 bits, taking the modulus for encoding payment IDs would be meaningless, so we don't.

[11] Transaction data does not indicate which output a payment ID 'belongs' to. Recipients have to identify their own payment IDs.

$$\begin{aligned}
k_{\text{mask}} &= \mathcal{H}_n(k_t^v rG, \text{pid\_tag}) \\
k_{\text{payment ID}} &= k_{\text{mask}} \rightarrow \text{reduced to bit length of payment ID} \\
\text{payment ID} &= \text{XOR}(k_{\text{payment ID}}, \text{encoded payment ID})
\end{aligned}$$

Similarly, senders can decode payment IDs they had previously encoded by recalculating the shared secret $k_{\text{mask}} = \mathcal{H}_n(rK_t^v)$.

5.5 Transaction types

Monero is a cryptocurrency under steady development. Transaction structures, protocols, and cryptographic schemes are always prone to evolving as new objectives or threats are found.

src/cryptonote_core/cryptonote_tx_utils.cpp construct_tx()

In this report we have focused our attention on *Ring Confidential Transactions*, a.k.a. *RingCT*, as they are implemented in the current version of Monero. RingCT is mandatory for all new Monero transactions, so we will not describe any deprecated transaction types, even if they are still partially supported.[12]

The transaction types we will describe in this chapter are `RCTTypeFull` and `RCTTypeSimple`. The former category (Section 5.6) closely follows the ideas explained by Shen Noether *at al.* in [61]. At the time that paper was written, the authors most likely intended to fully replace the original CryptoNote transaction scheme.

However, for multi-input transactions, the signature scheme formulated in that paper was thought to entail a risk on traceability. This will become clear when we supply technical details, but in short: if one spent output became identifiable, the rest of the spent outputs would also become identifiable. This would have an impact on the traceability of currency flows, not only for the transaction originator affected, but also for the rest of the blockchain.

To mitigate this risk, the Monero Research Lab decided to use a related, yet different signature scheme for multi-input transactions. The transaction type `RCTTypeSimple` (Section 5.7) is used in those situations. The main difference, as we will see later, is that each input is signed independently.

We present a conceptual summary of transactions in Section 5.8.

[12] RingCT was first implemented in January, 2017, (v4 of the protocol). It was made mandatory for all new transactions in September, 2017 (v6 of the protocol). [12]

5.6 Ring Confidential Transactions of type `RCTTypeFull`

src/ringct/rctSigs.cpp genRct()

By default, the current code base applies this type of signature scheme when transactions have only one input. The scheme itself allows multi-input transactions, but when it was introduced, the Monero Research Lab decided that it would be advisable to use it only on single-input transactions. For multi-input transactions, existing Monero wallets use the `RCTTypeSimple` scheme described later.

Our perception is that the decision to limit `RCTTypeFull` transactions to one input was rather hastily taken, and that it might change in the future, perhaps if the algorithm to select additional mix-in outputs is improved and ring sizes are increased. Also, Shen Noether's original description in [61] did not envision constraints of this type. At any rate, it is not a hard constraint. An alternative wallet might choose to sign transactions using either scheme, independently of the number of inputs involved.

We have therefore chosen to describe the scheme as if it were meant for multi-input transactions.

An actual example of a `RCTTypeFull` transaction, with all its components, can be inspected in Appendix A.

5.6.1 Amount commitments

Recall from Section 4.2 that we had defined a commitment to an output's amount b as:

$$C(b) = yG + bH$$

In the context of Monero, output recipients should be able to reconstruct the amount commitments. This means the blinding factor y and amount b must be communicated to the receiver.

The solution adopted in Monero is a Diffie-Hellman shared secret rK_B^v. For any given transaction in the blockchain, each of its outputs $t \in \{1, ..., p\}$ has two associated values called *mask* and *amount* satisfying[13]

$$mask_t = y_t + \mathcal{H}_n(\mathcal{H}_n(rK_B^v, t))$$
$$amount_t = b_t + \mathcal{H}_n(\mathcal{H}_n(\mathcal{H}_n(rK_B^v, t)))$$

src/ringct/rctOps.cpp ecdhEncode()

The receiver Bob will be able to calculate the blinding factor y_t and the amount b_t using the *transaction public key* rG and his *view key* k_B^v. He can also check that the commitment $C(y_t, b_t)$ provided in the transaction data, henceforth denoted C_t^b, corresponds to the amount at hand.

More generally, any third party with access to Bob's *view key* could decrypt his output amounts, and make sure they agree with their associated commitments.

[13] As with the one-time address K^o, the output index t is appended to the hash in each mask/amount pair. This ensures outputs directed to the same address do not have similar *masks* and *amounts*, except with negligible probability. *Yes*, that is three hashes for the amount. The extra hash ensures a different value is used for one-time addresses and the two ECDH exchange parts.

src/cryptonote_core/cryptonote_tx_utils.cpp construct_tx_with_tx_key() calls derivation_to_scalar()

5.6.2 Commitments to zero

Assume a transaction sender has previously received various outputs with amounts $a_1, ..., a_m$ addressed to one-time addresses $K^o_{\pi,1}, ..., K^o_{\pi,m}$ and with amount commitments $C^a_{\pi,1}, ..., C^a_{\pi,m}$.

This sender knows the private keys $k^o_{\pi,1}, ..., k^o_{\pi,m}$ corresponding to the one-time addresses (Section 5.2). The sender also knows the blinding factors x_j used in commitments $C^a_{\pi,j}$ (Section 5.6.1).

A transaction consists of inputs $a_1, ..., a_m$ and outputs $b_1, ..., b_p$ such that $\sum\limits_{j=1}^{m} a_j - \sum\limits_{t=1}^{p} b_t = 0$.

The sender re-uses the commitments from the previous outputs, $C^a_{\pi,1}, ..., C^a_{\pi,m}$, and creates commitments for $b_1, ..., b_p$. Let these new commitments be $C^b_1, ..., C^b_p$.

As hinted in Section 4.2, the sum of the commitments will not be 0, but a curve point zG:

$$\sum_j C^a_{\pi,j} - \sum_t C^b_{\pi,t} = zG$$

The sender will know z, allowing him to create a signature on this *commitment to zero*.

Indeed, z follows from the blinding factors if and only if input amounts equal output amounts (recalling Section 4.1, we don't know γ in $H = \gamma G$).

$$\begin{aligned}
&\sum_{j=1}^{m} C^a_{\pi,j} - \sum_{t=1}^{p} C^b_{\pi,t} \\
=&\sum_j x_j G - \sum_t y_t G + (\sum_j a_j - \sum_t b_t) H \\
=&\sum_j x_j G - \sum_t y_t G \\
=&zG
\end{aligned}$$

5.6.3 Signature

The sender selects v sets of size m, of additional unrelated addresses and their commitments from the blockchain, corresponding to apparently unspent outputs.[14] She mixes the addresses in a *ring* with her own m unspent outputs' addresses, adding false commitments to zero, as follows:

[14] In Monero it is standard for the set of 'additional unrelated addresses' to be selected from this distribution: 50% from 1.8 days ago up to the default transaction spendable age, which is usually 10 blocks ago, 50% from the remaining blockchain. Each segment has a triangle probability aimed at the 1.8 day mark, where an output 1.8 days old is twice as likely to be chosen as an output 0.9 days old. To spend an output of type X, we find all other outputs of type X (e.g. RingCT outputs) and choose its ring members from that set based on the distribution. The triangle probabilities are achieved by rolling a random number for each decoy ring member, normalizing it, taking the square root, multiplying it by the number of eligible type X outputs, and using the output at that index in the group (flip the index if the triangle faces backward by computing [#eligible - index]).

CRYPTO-NOTE_ DE-FAULT_TX _SPEND-ABLE_AGE src/wallet/ wallet2.cpp get_outs()

$$\begin{aligned}\mathcal{R} = \{&\{K^o_{1,1}, ..., K^o_{1,m}, (\sum_j C_{1,j} - \sum_t C^b_t)\},\\ &...\\ &\{K^o_{\pi,1}, ..., K^o_{\pi,m}, (\sum_j C^a_{\pi,j} - \sum_t C^b_t)\},\\ &...\\ &\{K^o_{v+1,1}, ..., K^o_{v+1,m}, (\sum_j C_{v+1,j} - \sum_t C^b_t)\}\}\end{aligned}$$

Looking at the structure of the key ring, we see that if

$$\sum_j C^a_{\pi,j} - \sum_t C^b_t = 0$$

then any observer would recognize the set of addresses $\{K^o_{\pi,1}, ..., K^o_{\pi,m}\}$ as the ones in use as inputs, and therefore currency flows would be traceable.

With this observation made we can see the utility of zG. All commitment terms in $\mathcal{R}$ return some EC point, and the π^{th} such term is zG. This allows us to create an MLSAG signature (Section 3.3) on $\mathcal{R}$.

MLSAG signature for inputs The private keys for $\{K^o_{\pi,1}, ..., K^o_{\pi,m}, (\sum_j C^a_{\pi,j} - \sum_t C^b_t)\}$ are $k^o_{\pi,1}, ..., k^o_{\pi,m}, z$, which are known to the sender. MLSAG in this scenario does not use a key image for the commitment to zero zG. This means building and verifying the signature excludes the term $r_{i,m+1}\mathcal{H}_p(K_{i,m+1}) + c_i\tilde{K}_z$.

src/ringct/ rctSigs.cpp prove-RctMG()

The message $\mathfrak{m}$ signed in the input MLSAG is essentially a hash of all transaction information *except* for the MLSAG signature itself.[15] This ensures transactions are tamper-proof from the perspective of both transaction authors and verifiers.

k^o is the essence of Monero's transaction model. Signing $\mathfrak{m}$ with k^o proves you are the owner/recipient of the amount committed to in C^a. Verifiers can be confident that transaction authors are spending their own funds.

Range proofs for outputs To avoid the amount ambiguity of outputs described in Section 4.3, the sender must also employ the Borromean signature scheme of Section 3.4 to sign amount ranges for each output $t \in \{1, ..., p\}$. No message $\mathfrak{m}$ is signed by the Borromean signatures.

src/ringct/ rctSigs.cpp prove-Range()

[15] The actual message is $\mathfrak{m} = \mathcal{H}(\mathcal{H}(tx\_prefix), \mathcal{H}(ss), \mathcal{H}(\text{range proof signatures}))$ where:
tx_prefix ={transaction era version (i.e. ringCT = 2), inputs {key offsets, key image}, outputs {one-time addresses}, extra {transaction public key, payment ID or encoded payment ID, misc.}}
ss ={signature type (simple vs full), transaction fee, pseudo output commitments for inputs, ecdhInfo (masks and amounts), output commitments}.
See Appendices A & B regarding this terminology.

src/ringct/ rctSigs.cpp get_pre_mlsag _hash()

Range proofs are not needed for input amounts because they are either expressed clearly (as with transaction fees and block rewards), or were proven in range when first created as outputs.

In the current version of the Monero software, each amount is expressed as a fixed point number of 64 bits. This means the data for each range proof will contain 64 bit commitments and $2 \cdot 64 + 1$ signature terms.

5.6.4 Transaction fees

Typically transaction outputs are *lower* in total than transaction inputs, in order to provide a fee that will incentivize miners to include the transaction in the blockchain.[16] Transaction fee amounts are stored in clear text in the transaction data transmitted to the network. Miners can create an additional output for themselves with the fee. This fee amount must be converted into a commitment so verifiers can confirm transactions sum to zero.

The solution is to calculate the commitment of the fee f without the masking effect of any blinding factor. That is, $C(f) = fH$, where f is communicated in clear text.

The network verifies the MLSAG signature on $\mathcal{R}$ by including fH as follows:

$$(\sum_j C_{i,j} - \sum_t C_t^b) - fH$$

Which works because this is a commitment to zero:

$$(\sum_j C_{\pi,j} - \sum_t C_t^b) - fH = zG$$

5.6.5 Avoiding double-spending

An MLSAG signature (Section 3.3) contains images $\tilde{K}_j$ of private keys $k_{\pi,j}$. An important property in any cryptographic signature scheme is that it should be unforgeable with non-negligible probability. Therefore, to all practical effects, we can assume a signature's key images must have been deterministically produced from legitimate private keys.

src/cryptonote_core/blockchain.cpp have_tx_keyimges_as_spent()

The network need only verify that key images included in MLSAG signatures (corresponding to inputs and calculated as $\tilde{K}_j^o = k_{\pi,j}^o \mathcal{H}_p(K_{\pi,j}^o)$) have not appeared before in other transactions.[17] If they have, then we can be sure we are witnessing an attempt to re-spend an output $C_{\pi,j}^a$ addressed to $K_{\pi,j}^o$.

[16] In Monero there is a minimum base fee per kB of transaction data. It is semi-mandatory because while you can create new blocks with tiny-fee transactions, most Monero nodes won't relay such transactions to other nodes. We go into more detail on this in Section 6.3.3.

[17] Verifiers must also check the key image is a member of the generator's subgroup (recall Section 3.1).

If someone tries to spend $C^a_{\pi,j}$ twice, they will reveal the index π for both transactions where it appears. This has two effects: 1) all outputs at index π in the first transaction are revealed as its real inputs, and 2) all outputs at index π in the second transaction are revealed as not having been spent before. The second is a problem even considering miners would reject the double-spend transaction.

These effects could weaken the network benefits of ring signatures, and are part of the reason `RCTTypeFull` is only used for single-input transactions. The other main reason is that a cryptanalyst would know that, in general, all real inputs share an index.

5.6.6 Space and verification requirements

MLSAG signature (inputs)

From Section 3.3 we recall that an MLSAG signature in this context would be expressed as

$$\sigma(\mathfrak{m}) = (c_1, r_{1,1}, ..., r_{1,m+1}, ..., r_{v+1,1}, ..., r_{v+1,m+1}) \text{ with } (\tilde{K}^o_1, ..., \tilde{K}^o_m)$$

As a legacy of CryptoNote, the values $\tilde{K}^o_j$ are not referred to as part of the signature, but rather as *images* of the private keys $k^o_{\pi,j}$. These *key images* are normally stored separately in the transaction structure, as they are used to detect double-spending attacks.

With this in mind and assuming point compression, an MLSAG signature will require $((v+1)\cdot(m+1)+1)\cdot 32$ bytes of storage, where v is the mixin level and m is the number of inputs. In other words, a transaction with 1 input and a total ring size of 32 would consume $(32\cdot 2+1)\cdot 32 = 2080$ bytes.

To this value we would add 32 bytes to store the key image of each input, for $m\cdot 32$ bytes of storage, and additional space to store the ring member offsets in the blockchain (see Appendix A). These offsets are used by verifiers to find each MLSAG signature's ring members' output keys and commitments in the blockchain, and are stored as variable length integers, hence we can not exactly quantify the space needed.[18]

Including the computation of $(\sum_j C_{i,j} - \sum_t C^b_t) - fH$, and verifying key images are in G's subgroup with $l\tilde{K} \overset{?}{=} 0$, we verify a `RCTTypeFull` transaction's MLSAG (Section 3.3) with:

src/ringct/ rctSigs.cpp verRctMG()

PA	Point addition for some points A, B: $A+B$ $[m*(v+1)]$
PS	Point subtraction for some points A, B : $A-B$ $[(v+1)*(p+1)]$
VBSM	Variable-base scalar multiplications for some integer a, and point P: aP $[m]$
KBSM	Known-base scalar multiplications for some integer a: aG $[1]$
DVBA	Double-variable-base addition for some integers a, b, and point B: $aG+bB$ $[(m+1)*(v+1)]$
VBA	Variable-base additions for some integers a, b, and points A, B: $aA+bB$ $[m*(v+1)]$

[18] Imagine the blockchain contains a long list of transaction outputs. We report the indices of outputs we want to spend. Now, bigger indexes require more storage space. We just need the 'absolute' position of *one* index, and the 'relative' position of the other indices. For example, with real indices {7,11,15,20} we just need to report {7,4,4,5}. Verifiers can compute the last index like (7+4+4+5 = 20).

src/cryptonote_basic/ cryptonote_format_utils.cpp absolute_output_offsets_to_relative()

Range proofs (outputs)

From Section 3.4, Section 4.3, and Section 5.6.3, we know that a Monero Borromean signature for range proofs takes the form of an n-tuple

src/ringct/ringCT.cpp genBorromean()

$$\sigma = (c_1, r_{0,1}, r_{0,2}, r_{1,1} ..., r_{63,2})$$

Ring keys are considered part of ring signatures. However, in this case it is only necessary to store the commitments C_i, as the ring key counterparts $C_i - 2^i H$ can be easily derived (for verification purposes).

Respecting this convention, a range proof will require $(1 + 64 \cdot 2 + 64)32 = 6176$ bytes per output.

Including $C_i - 2^i H$ and checking $\sum_i C_i = C$, range proofs for p outputs will require this to verify (Section 4.4):

src/ringct/rctSigs.cpp verRange()

VRSF Verify range proof special function $[p * 64]$
DVBA Double-variable-base addition for some integers a, b, and point B: $aG + bB$ $[p * 2 * 64]$

5.7 Ring Confidential Transactions of type `RCTTypeSimple`

src/ringct/rctSigs.cpp genRctSimple()

In the current Monero code base, transactions with more than one input are signed using a scheme known as `RCTTypeSimple`.

The main characteristic of this approach is that instead of signing the entire set of inputs at once, the sender signs each input separately.

Among other things, this means we can't use commitments to zero in the same way as for `RCTTypeFull` transactions. It's really unlikely that we could match an input amount to an output amount, and in most cases each individual input will be less than the sum of outputs. So, we cannot directly commit *inputs* − *outputs* = 0.

In more detail, assume that Alice wants to sign input j. Imagine for a moment we could sign an expression like this

$$C_j^a - \sum_t C_t^b = (x_j - \sum_t y_t)G + (a_j - \sum_t b_t)H$$

Since $a_j - \sum\limits_t b_t \neq 0$, Alice would have to solve the DLP for $H = \gamma G$ in order to obtain the private key of the expression, something we have assumed to be computationally difficult.

5.7.1 Amount commitments

As explained, if inputs are decoupled from each other then the sender can't sign an aggregate commitment to zero. On the other hand, signing each input individually implies an intermediate approach. The sender could create new commitments to the input amounts and commit to zero with respect to each of the previous outputs being spent. In this way, the sender could prove the transaction takes as input only the outputs of previous transactions.

In other words, assume the amounts being spent are $a_1, ..., a_m$. These amounts were outputs in previous transactions, in which they had commitments

$$C_j^a = x_j G + a_j H$$

The sender can create new commitments to the same amounts but using different blinding factors; that is,

$$C_j'^a = x_j' G + a_j H$$

Clearly, she would know the private key of the difference between the two commitments:

$$C_j^a - C_j'^a = (x_j - x_j')G$$

Hence, she would be able to use this value as a *commitment to zero* for each input. Let us say $(x_j - x_j') = z_j$, and call each $C_j'^a$ a *pseudo output commitment*.

Similarly to `RCTTypeFull` transactions, the sender can include each output's encoded blinding factor (mask) for y_t and amount for b_t in the transaction (see Section 5.6.1), which will allow each receiver t to decode y_t and b_t using the shared secret rK_t^v.

Before committing a transaction to the blockchain, the network will want to verify that the transaction balances. In the case of `RCTTypeFull` transactions, this was simple, as the MLSAG signature scheme implies each sender has signed with the private key of a commitment to zero.

For `RCTTypeSimple` transactions, blinding factors for input and output commitments are selected such that

$$\sum_j x_j' - \sum_t y_t = 0$$

This allows us to prove input amounts equal output amounts:

src/ringct/rctSigs.cpp verRct-Simple()

$$(\sum_j C_j'^a - \sum_t C_t^b) - fH = 0$$

Fortunately, choosing such blinding factors is simple. In the current version of Monero, all blinding factors are random except x_m', which is simply set to

genRct-Simple()

$$x_m' = \sum_t y_t - \sum_{j=1}^{m-1} x_j'$$

5.7.2 Signature

As we mentioned, in transactions of type `RCTTypeSimple` each input is signed individually. We use the same MLSAG signature scheme as for `RCTTypeFull` transactions, except with different signing keys.

src/ringct/rctSigs.cpp proveRctMGSimple()

Assume that Alice is signing input j. This input spends a previous output with key $K^o_{\pi,j}$ that had commitment $C^a_{\pi,j}$. Let $C'^a_{\pi,j}$ be a new commitment for the same amount but with a different blinding factor.

Similar to the previous scheme, the sender selects v unrelated outputs and their respective commitments from the blockchain to mix with the real, j^{th}, input

$$K^o_{1,j}, ..., K^o_{\pi-1,j}, K^o_{\pi+1,j}, ..., K^o_{v+1,j}$$
$$C_{1,j}, ..., C_{\pi-1,j}, C_{\pi+1,j}, ..., C_{v+1,j}$$

She can sign input j using the following ring:

$$\begin{aligned}\mathcal{R}_j = \{&\{K^o_{1,j}, (C_{1,j} - C'^a_{\pi,j})\},\\ &...\\ &\{K^o_{\pi,j}, (C^a_{\pi,j} - C'^a_{\pi,j})\},\\ &...\\ &\{K^o_{v+1,j}, (C_{v+1,j} - C'^a_{\pi,j})\}\}\end{aligned}$$

Alice will know the private keys $k^o_{\pi,j}$ for $K^o_{\pi,j}$, and z_j for the commitment to zero ($C^a_{\pi,j}$ - $C'^a_{\pi,j}$). Recalling Section 5.6.3, there is no key image for the commitments to zero z_jG, and consequently no corresponding key image component in each input's signature's construction.

Each input in an `RCTTypeSimple` transaction is signed individually, applying the scheme described in Section 5.6.3, but using rings like $\mathcal{R}_j$ as defined above.

The advantage of signing inputs individually is that the set of real inputs and commitments to zero need not be placed at the same index π, as they are in the aggregated case. This means even if one input's origin became identifiable, the other inputs' origins would not.

The message $\mathfrak{m}$ signed by each input is essentially the same as for `RCTTypeFull` transactions (see Footnote 15), except it includes pseudo output commitments for the inputs. Only one message is produced, and each input MLSAG signs it.

5.7.3 Space and verification requirements

MLSAG signature (inputs)

Each ring $\mathcal{R}_j$ contains $(v+1)\cdot 2$ keys. Using the point compression technique from Section 2.4.2, an input signature σ will require $(2(v+1)+1)\cdot 32$ bytes. On top of this is, the key image $\tilde{K}^o_{\pi,j}$ and the pseudo output commitment $C'^a_{\pi,j}$ leave a total of $(2(v+1)+3)\cdot 32$ bytes per input.

A transaction with 20 inputs using rings with 32 total members will need $((32 \cdot 2 + 3) \cdot 32)20 = 42880$ bytes.

For the sake of comparison, if we were to apply the `RCTTypeFull` scheme to the same transaction, the MLSAG signature and key images would require $(32 \cdot 21 + 1) \cdot 32 + 20 \cdot 32 = 22176$ bytes.

Including the computation of $(C_{i,j} - C'^{a}_{\pi,j})$ and $(\sum_j C'^{a}_j \overset{?}{=} \sum_t C^b_t + fH)$, and verifying key images are in G's subgroup with $l\tilde{K}$, we verify all of a `RCTTypeSimple` transaction's MLSAGs with:

src/ringct/rctSigs.cpp verRctMGSimple() verRctSimple()

PA	Point addition for some points A, B: $A + B$ $[m + p + 1]$
PS	Point subtraction for some points A, B : $A - B$ $[m * (v + 1)]$
VBSM	Variable-base scalar multiplications for some integer a, and point P: aP $[m]$
KBSM	Known-base scalar multiplications for some integer a: aG $[1]$
DVBA	Double-variable-base addition for some integers a, b, and point B: $aG + bB$ $[m * 2 * (v + 1)]$
VBA	Variable-base additions for some integers a, b, and points A, B: $aA + bB$ $[m * (v + 1)]$

Range proofs (outputs)

The size of range proofs remains the same for `RCTTypeSimple` transactions. As we calculated for `RCTTypeFull` transactions, each output will require 6176 bytes of storage.

As before, each range proof will require this to verify (Section 4.4):

src/ringct/rctSigs.cpp verRange()

VRSF	Verify range proof special function $[p * 64]$
DVBA	Double-variable-base addition for some integers a, b, and point B: $aG + bB$ $[p * 2 * 64]$

5.8 Concept summary: Monero transactions

To summarize this chapter we present the main content of a transaction, organized for conceptual clarity. Real examples can be found in Appendices A and B.

- Type: '0' is `RCTTpeNull` (for miners), '1' is `RCTTypeFull`, and '2' is `RCTTypeSimple`
- Inputs: for each input $j \in \{1, ..., m\}$ spent by the transaction author
  - **Ring member offsets**: a list of 'offsets' indicating where a verifier can find input j's ring members $i \in \{1, ..., v+1\}$ in the blockchain (includes the real input)
  - **MLSAG Signature**: σ terms c_1, and $r_{i,j}$ & $r^z_{i,j}$ for $i \in \{1, ..., v+1\}$ and input j
  - **Key image**: the key image $\tilde{K}^{o,a}_j$ for input j
  - **Pseudo output commitment** [`RCTTypeSimple` only]: C'^a_j for input j
- Outputs: for each output $t \in \{1, ..., p\}$ to address or subaddress (K^v_t, K^s_t)
  - **One-time address**: $K^{o,b}_t$ for output t
  - **Output commitment**: C^b_t for output t
  - **Diffie-Hellman terms**: so receivers can compute C^b_t and b_t for output t
    - *Mask*: $y_t + \mathcal{H}_n(\mathcal{H}_n(rK^v_t, t))$
    - *Amount*: $b_t + \mathcal{H}_n(\mathcal{H}_n(\mathcal{H}_n(rK^v_t, t)))$
  - **Range proof** for output amount b_t using a Borromean ring signature
    - *Signatures*: σ terms c_1, and $r_{i,j}$ for $i \in \{0, ..., 63\}$ and $j \in \{1, 2\}$
    - *Bit commitments*: C_i for $i \in \{0, ..., 63\}$
- Transaction fee: communicated in clear text multiplied by 10^{12} (i.e. atomic units, see chapter 6), so a fee of 1.0 would be recorded as 1000000000000
- Extra: includes the transaction public key rG, or, if at least one output is directed to a subaddress, $r_t K^{s,i}_t$ for each subaddress'd output t and $r_t G$ for each normal address'd output t, and a payment ID or encoded payment ID (max one per transaction)[19]

[19] No information stored in the 'extra' field is verified, though it *is* signed by input MLSAGs, so no tampering is possible (except with negligible probability).

5.8.1 Storage and verification requirements

For RCTTypeFull we need $((v+1)\cdot(m+1)+1)\cdot 32$ bytes of storage, and verify with: src/ringct/rctOps.cpp

PA Point addition for some points A, B: $A + B$ $[m*(v+1)]$
PS Point subtraction for some points A, B : $A - B$ $[(v+1)*(p+1)]$
KBSM Known-base scalar multiplications for some integer a: aG [1]
DVBA Double-variable-base addition for some integers a, b, and point B: $aG + bB$ $[(m+1)*(v+1)]$
VBA Variable-base additions for some integers a, b, and points A, B: $aA + bB$ $[m*(v+1)]$

For RCTTypeSimple we need $(2(v+1)+2)\cdot m\cdot 32$ bytes of storage, and verify with:

PA Point addition for some points A, B: $A + B$ $[m+p+1]$
PS Point subtraction for some points A, B : $A - B$ $[m*(v+1)]$
KBSM Known-base scalar multiplications for some integer a: aG [1]
DVBA Double-variable-base addition for some integers a, b, and point B: $aG + bB$ $[m*2*(v+1)]$
VBA Variable-base additions for some integers a, b, and points A, B: $aA + bB$ $[m*(v+1)]$

Output range proofs need $p \cdot 6176$ bytes of storage, and verify with:

VRSF Verify range proof special function $[p*64]$
DVBA Double-variable-base addition for some integers a, b, and point B: $aG + bB$ $[p*2*64]$

Miscellaneous requirements:

- Input key images: $m*32$ bytes, and verifies with:

VBSM Variable-base scalar multiplications for some integer a, and point P: aP $[m]$

- One-time output addresses: $p*32$ bytes
- Output commitments: $p*32$ bytes
- Diffie-Hellman terms for outputs: $2*p*32$ bytes
- Transaction public key: 32 bytes normally, $p*32$ bytes if sending to at least one subaddress.
- Payment ID: 8 bytes for an integrated address, or 32 bytes for a plain payment ID. Max one per tx.
- Transaction fee: stored as variable length integer[20], so ≤ 64 bits
- Input offsets: stored as variable length integer, so ≤ 64 bits per input, for m inputs

[20] See [16] for an explanation of Monero's varint data type.

CHAPTER 6

The Monero Blockchain

The Internet Age has brought a new dimension to the human experience. We can correspond with people on every corner of the planet, and an unimaginable wealth of information is at our fingertips. Exchanging goods and services is fundamental to a peaceful and prosperous society, and in the digital realm we can offer our productivity to the whole world.

Media of exchange (moneys) are essential, giving us a point of reference to an immense diversity of economic goods that would otherwise be impossible to evaluate, and enabling mutually beneficial interactions between people with nothing in common. Throughout history there have been many kinds of money, from sea shells to paper to gold. Those were exchanged by hand, and now money can be exchanged electronically.

In the current, by far most pervasive, model, electronic transactions are handled by third party financial institutions. These institutions are given custody of money and trusted to transfer it upon request. Such institutions must mediate disputes, their payments are reversible, and they can be censored or controlled by powerful organizations. [49]

It seems appropriate for the Internet Age to have its own unique currencies.

Note: this chapter includes more implementation details than previous chapters, as a blockchain's nature depends heavily on its parameters and specific structure.

6.1 Digital currency

Let's try to make a digital currency from scratch.

Suppose two pals Jim and Dwight need a currency for their Secret Society of Stealthy Sleuths. Jim sends an email to Dwight saying "As co-founder, I hereby conjure 5 Stealthbucks for myself and 5 Stealthbucks for you." Later that day Dwight discovers Kevin had made 69 Stealthbucks for himself and used them to buy Dwight's bobblehead from Jim. There should only be 10 Stealthbucks, what went wrong?

In the **email model** anyone can make Stealthbucks, and anyone can send their Stealthbucks over and over. It does not have a limited supply, nor is it 'double spend proof'.

Jim proposes a new system, Stealthbucks 2.0, where he keeps a record on his computer of who owns all the Stealthbucks. To exchange them, people need to talk to Jim. Since Jim is the sole custodian, there is no question the Stealthbucks on Kevin's computer are bogus. Jim promises Dwight they will start off with 100 Stealthbucks each and that's it - no more. Dwight tries to buy his bobblehead back and Jim says "You need to prove your identity!" What went wrong?

In the **video game model**, where the entire currency is stored on one central database, users rely on the custodian to be honest. The currency's supply is unverifiable for observers, and the custodian can change the rules at any time, or be censored by powerful outsiders.

6.1.1 Shared version of events

Since Jim can't be trusted to manage Stealthbucks, Dwight suggests setting up a bunch of computers that each have a record of every Stealthbuck transaction. When a new transaction is made on one computer, it is broadcast to the other computers, which only accept it if it follows the rules.

Rule 1: Money can only be created in clearly defined scenarios.

Rule 2: Transactions spend money that already exists.

Rule 3: Transactions output money equal to the money spent.

Rule 4: Transactions are formatted correctly.

Rule 5: Only the person who owns a piece of money can spend it.

Rule 6: A person can only spend a piece of money once.

Rules 2-6 are already covered by the transaction schemes discussed in Chapter 5, which add the benefits of ambiguous signing, anonymous receipt of funds, and unreadable amount transfers. We explain Rule 1 later in this chapter.

What if one of the computers goes rogue and starts making a bunch of Stealthbucks for itself? Users only benefit from Stealthbucks when other users accept it in exchange, so no one will accept transactions spending counterfeit Stealthbucks if they expect *other* users only want legitimate coins. Each user must act honestly and follow the same rules as everyone else if they want to spend their money. We call this situation a 'social minima', 'Schelling point', or 'social contract'.

Simply storing all transactions has a problem. If two computers receive legitimate transactions spending the same money, before they have a chance to send the information to each other, how do they decide which is correct? There is a 'fork' in the currency, because two different copies that follow the same rules exist.

It seems obvious the earliest legitimate transaction spending a piece of money should be canonical. This is easier said than done. As we will see, obtaining consensus for transaction histories constitutes the raison d'être of blockchain technology.

6.1.2 Simple blockchain

First we need all of the computers, henceforth referred to as *nodes*, to agree on the order of transactions.

Let's say Stealthbucks started with a 'genesis' declaration by Jim and Dwight: "Let the Stealthbucks begin!". We call this message a 'block', and its block hash is

$$BH_G = \mathcal{H}(\text{Let the Stealthbucks begin!})$$

Every time a node receives some transactions, they use those transactions' hashes, TH, as messages, along with the previous block's hash, and compute new block hashes

$$\begin{aligned} BH_1 &= \mathcal{H}(BH_G, TH_1, TH_2, ...) \\ BH_2 &= \mathcal{H}(BH_1, TH_1, TH_2, ...) \end{aligned}$$

And so on, publishing each new block of messages as it's made. Each new block references the previous, most recently published block. In this way a clear order of events extends all the way back to the genesis message. We have a chain of blocks: a very simple 'blockchain'.[1]

Nodes can include a timestamp in their blocks to aid record keeping. If most nodes are honest with timestamps then the blockchain provides a decent picture of when each transaction was recorded.

What if different transactions spending the same money are added to blocks referencing the same previous block, which are published at the same time? The network of nodes will fork again, as each node receives one of the new blocks before the other (for simplicity, imagine about half the nodes have each side of the fork). Let's keep improving our blockchain.

6.2 Difficulty

If nodes can publish new blocks whenever they want, the network will tend to fracture and diverge into many different, equally legitimate, chains. Say it takes 30 seconds to make sure everyone in

[1] A blockchain is technically a 'directed acyclic graph' (DAG), with Bitcoin-style blockchains a one-dimensional variant. DAGs contain a finite number of nodes and one-directional edges (vectors) connecting nodes. If you start at one node, you will never loop back to it no matter what path you take. [5]

the network gets a new block. What if transactions are made every 31 seconds? New blocks would just barely make it everywhere before another one gets sent out.

Now what if new blocks are every 15 seconds, 10 seconds, etc? Since message transmission time is a function of distance, the network would fracture into small clumps circumscribed by the time it takes for a new block to propagate before a new one is produced.

We can avoid this by controlling how fast the entire network makes new blocks. If the time it takes to make a new block is much higher than the time for the previous block to reach every node, the network will tend to remain intact.

6.2.1 Mining a block

The output of a cryptographic hash function is uniformly distributed. This means, for any given input, its hash is equally likely to be every single possible output. Furthermore, it takes a certain amount of time to compute a single hash.

Let's imagine a hash function $\mathcal{H}_i(x)$ which outputs a number from 1 to 100: $\mathcal{H}_i(x) \in^D_R \{1, ..., 100\}$. We use $\in^D_R$ to say the output is deterministically random. Given some x, $\mathcal{H}_i(x)$ selects the same 'random' number from {1,...,100} every time you calculate it. It takes 1 minute to calculate $\mathcal{H}_i(x)$.

Say we are given a message $\mathfrak{m}$ and so-called 'nonce' $n = 1$, and told to find an n such that $\mathcal{H}_i(\mathfrak{m}, n)$ outputs a number less than or equal to the *target* $t = 10$ (i.e. $\mathcal{H}_i(\mathfrak{m}, n) \in \{1, ..., 10\}$). Guessing and checking by incrementing n for each new hash, how many hashes will it probably take?

It should be around 10 hashes, for 10 minutes of hashing, because there is only a $1/10^{\text{th}}$ chance any given input will output a good answer. This isn't to say $n = 10$ is the right answer, just that if we take a lot of messages and do this process for each one, $n = 10$ will be the *average* value.

We call searching for a useful nonce *mining*, and publishing the message with its nonce is a *proof of work* because it proves we looked for a useful nonce (even if we were lucky and it needed just one hash to find), which anyone can verify by computing $\mathcal{H}_i(\mathfrak{m}, n)$.

Now say we have a hash function for generating proofs of work, $\mathcal{H}_{PoW} \in^D_R \{0, ..., m\}$, where m is its maximum possible output. Given a message $\mathfrak{m}$ (a block of information), a nonce n to mine, and a target t, we can define the *difficulty* d, or expected number of hashes, like this: $d = m/t$. If $\mathcal{H}_{PoW}(\mathfrak{m}, n) * d \leq m$, then n is accepted.

src/crypto-note_basic/ difficulty.cpp check_hash()

As the target gets smaller, the difficulty rises and it takes a computer more and more hashes, and therefore longer and longer periods of time, to find useful nonces.

6.2.2 Mining speed

Assume all nodes are mining nonces at the same time, but quit on their 'current' block when they receive a new one from the network. They immediately start mining a fresh block that references the new one.

Suppose we collect a bunch b of blocks from the blockchain (say, with index $u \in \{1, ..., b\}$) which each had a difficulty d_u. For now, assume the nodes who mined them were honest, so each block timestamp TS_u is accurate. The total time between the earliest block and most recent block is $totalTime = TS_b - TS_1$. The approximate number of hashes to mine all the blocks is $totalDifficulty = \sum_u d_u$.

Now we can guess how fast the network, with all its nodes, can compute hashes. If the actual speed didn't change much while the bunch of blocks was being produced, it should be effectively[2]

$$hashSpeed \approx totalDifficulty/totalTime$$

If we want to set the target time to mine new blocks, so blocks are produced at a rate (one block)/(target time), then from the hash speed we can calculate how many hashes it should take for the network to spend that amount of time mining.

$$miningHashes = hashSpeed * targetTime$$

Since difficulty is approximately how many hashes it takes to generate a proof of work, we can set the new difficulty equal to $miningHashes$. Note: we round up so difficulty never equals zero.

$$newDifficulty = (totalDifficulty/totalTime) * targetTime$$

There is no guarantee the next block will take $newDifficulty$ amount of hashes to mine, but over time and many blocks and constantly re-calibrating, the difficulty will track with the network's real hash speed and blocks will tend to take $targetTime$.[3]

6.2.3 Consensus: largest cumulative difficulty

Now we have a mechanism to resolve conflicts between chain forks. Since difficulty represents how much work was spent to mine a block, higher difficulty means more work performed.

By convention, the chain with highest cumulative difficulty (from all blocks in the chain), and therefore with most work spent constructing, is considered the real, legitimate version. If a chain splits and each fork has the same cumulative difficulty, nodes continue mining on their fork until one branch gets ahead of the other, at which point the weaker branch is discarded ('orphaned').

If nodes wish to change or upgrade the basic protocol, i.e. the set of rules a node considers when deciding if a blockchain copy or new block is legitimate, they may easily do so by forking the chain. Whether the new branch has any impact on users depends on how many nodes switch and how much software infrastructure is modified.[4]

[2] If node 1 tries nonce $n = 23$ and later node 2 also tries $n = 23$, node 2's effort is wasted because the network already 'knows' $n = 23$ doesn't work (otherwise node 1 would have published that block). The network's *effective* hash rate depends on how fast it hashes *unique* nonces for a given block of messages. As we will see, since miners include a miner transaction with one-time address $K^o \in^E_R \mathbb{Z}_l$ (E = effectively) in their blocks, blocks are always unique between miners except with negligible probability.

[3] If we assume network hash rate is constantly, gradually, increasing, then since new difficulties depend on *past* hashes (i.e. before the hash rate increased a tiny bit) we should expect actual block times to, on average, be slightly less than $targetTime$. The effect of this on the emission schedule (Section 6.3.1) could be canceled out by penalties from increasing block sizes, which we explore in Section 6.3.2.

[4] Monero has successfully changed its protocol 7 times, with nearly all users and miners adopting the changes each time. v1 April 18, 2014 [63]; v2 March, 2016; v3 September, 2016; v4 January, 2017; v5 April, 2017; v6 September, 2017; v7 April, 2018; see src/cryptonote_core/blockchain.cpp mainnet_hard_forks

For an attacker to convince honest nodes to alter the transaction history, perhaps in order to respend/unspend funds, he must create a chain fork (on the current protocol) with higher total difficulty than the main chain (which meanwhile continues to grow). This is very hard to do unless you control over 50% of the network hash speed and can outwork other miners. [49]

6.2.4 Mining in Monero

To make sure chain forks are on an even footing, we don't sample the most recent blocks (for calculating new difficulties), instead lagging our bunch b by l. For example, if there are 29 blocks in the chain (blocks 1,...,29), $b = 10$, and $l = 5$, we sample blocks 15-24 in order to compute block 30's difficulty.

If mining nodes are dishonest they can manipulate timestamps so new difficulties don't match the network's real hash speed. We get around this by sorting timestamps chronologically, then chopping off the first o outliers and last o outliers. Now we have a 'window' of blocks $w = b - 2 * o$. From the previous example, if $o = 3$ and timestamps are honest then we would chop blocks 15-17 and 22-24, leaving blocks 18-21 to compute block 30's difficulty from.

Monero is somewhat bizarre. Instead of sorting block difficulties so they correspond with their blocks' sorted timestamps, we use an array of the original bunch's blocks' cumulative difficulties, leave it unsorted, and chop off the o outliers from that. Cumulative difficulty for a block is that block's difficulty plus the difficulty of all previous blocks in the chain.

src/cryptonote_core_blockchain.cpp get_difficulty_for_next_block()

Using the chopped arrays of w timestamps and cumulative difficulties (indexed from 1,...,w), we define

$$totalTime = choppedSortedTimestamps[w] - choppedSortedTimestamps[1]$$
$$totalDifficulty = choppedCumulativeDifficulties[w] - choppedCumulativeDifficulties[1]$$

In Monero the target time is 120 seconds (2 minutes), $l = 15$ (30 mins), $b = 720$ (one day), and $o = 60$ (2 hours).[5]

src/cryptonote_config.h

Block difficulties are not stored in the blockchain, so someone downloading a copy of the blockchain and verifying all blocks are legitimate needs to recalculate difficulties from recorded timestamps. There are a few rules to consider for the first $b + l = 735$ blocks.

src/cryptonote_basic_difficulty.cpp next_difficulty()

Rule 1: Ignore the genesis block (block 0, with $d = 1$) completely. Blocks 1 and 2 have $d = 1$.

Rule 2: Before chopping off outliers, try to get the window w to compute totals from.

Rule 3: After w blocks, chop off high and low outliers, scaling the amount chopped until b blocks. If the amount of previous blocks (minus w) is odd, remove one more low outlier than high.

Rule 4: After b blocks, sample the earliest b blocks until $b+l$ blocks, after which everything proceeds normally - lagging by l.

[5] In March, 2016, (v2 of the protocol) Monero changed from 1 minute target block times to 2 minute target block times [11]. Other difficulty parameters have always been the same.

Monero proof of work (PoW)

Monero uses a proof of work hash algorithm known as Cryptonight, designed to be relatively inefficient on GPU, FPGA, and ASIC architectures [60] compared to standard hash functions like SHA256. In April, 2018, (v7 of the protocol) it was slightly modified to counter the advent of Cryptonight ASICs [22]. src/crypto/ slow-hash.c

6.3 Money supply

Obviously a digital currency needs a supply of money for users to transact with. There are two basic mechanisms for creating money in a blockchain-based cryptographic currency (a.k.a. cryptocurrency).

First, the currency's creators can simply conjure a set amount and distribute it to people in the genesis message. This is often called an 'airdrop'. Sometimes cryptocurrency creators give themselves a large amount of money in a so-called 'pre-mine'. [17]

Second, the currency can be automatically distributed as reward for mining a block, much like mining for gold. There are two types here. In the Bitcoin model the total possible supply is capped. Block rewards slowly decline to zero, after which no more money is ever made. In the inflation model the supply continues to rise indefinitely.

Some cryptocurrencies employ both mechanisms for money creation. In fact, Monero is based on a currency known as Bytecoin that had a large pre-mine, followed by block rewards [10]. Monero had no pre-mine, and as we will see, its block rewards slowly decline to a small amount after which all new blocks reward that same amount, making Monero an inflationary currency.

6.3.1 Block reward

The block reward concept is straightforward. Block miners, before mining for a nonce, make a 'miner transaction' with no inputs and one output. The output amount is equal to the block reward, plus transaction fees from all transactions to be included in the block, and is communicated in clear text. Nodes who receive a mined block must verify the block reward is correct, and can calculate the current money supply by summing all past block rewards together.

Besides distributing money, block rewards incentivize mining. If there were no block rewards (and no other mechanism), why would anyone mine new blocks? Perhaps altruism or curiosity. However, few miners makes it easy for a malicious actor to assemble >50% of the network's hash rate, with which they can easily rewrite recent chain history.[6]

With block rewards, competition between miners drives total hash rate up until the marginal cost of adding more hash rate is higher than the marginal reward of obtaining that proportion of mined

[6] As an attacker gets higher shares (beyond 50%) of the hash rate, it becomes easier and easier to rewrite older and older blocks.

blocks (which appear at a constant rate) (plus some premiums like risk and opportunity cost). This means as a cryptocurrency becomes more valuable, its total hash rate will increase and it becomes progressively more difficult and expensive to gather >50%.

Bit shifting

Bit shifting is used for calculating the base block reward (as we will see in Section 6.3.2, the block reward can sometimes be reduced below the base amount).

Suppose we have an integer A = 13, which has a bit representation [1101]. If we shift the bits of A down by 2, denoted A >> 2, we get [0011].01, which equals 3.25. In reality that last .01 outside the array, equal to 0.25, gets thrown away - 'shifted' into oblivion, leaving us with [0011] = 3.

This operation is equivalent to *floor*(13/4) = 3, or *floor*(A/2ˆ2), where *floor* rounds the number down - chopping off the 01 part of [0011].01.

Calculating base block reward for Monero

Let's call the current total money supply M, and the 'limit' of the money supply $L = 2^{64} - 1$ (in binary it is [11....11], with 64 bits). In the beginning of Monero, the base block reward B = (L-M) >> 20, or in other words *floor*((L-M)/2ˆ20). If M = 0, then, in decimal format,

$$\mathrm{L} = 18,446,744,073,709,551,615$$
$$\mathrm{B}_0 = (L-0) >> 20 = 17,592,186,044,415$$

These numbers are in 'atomic units' - 1 atomic unit of Monero can't be divided (there is no 0.5 atomic units). Clearly atomic units are ridiculous - L is over 18 quintillion! We can divide everything by 10ˆ12 to move the decimal point over, giving us the standard units of Monero (a.k.a. XMR, Monero's so-called 'stock ticker').

$$\frac{\mathrm{L}}{10^{12}} = 18,446,744.073709551615$$
$$\mathrm{B}_0 = \frac{(L-0) >> 20}{10^{12}} = 17.592186044415$$

And there it is, the very first block reward, dispersed to pseudonymous thankful_for_today (who was responsible for starting the Monero project) in Monero's genesis block [63], was about 17.6 Moneroj! See Appendix D to confirm this for yourself.[7]

As more and more blocks are mined block rewards accumulate and M grows, continuously lowering future block rewards. Initially (since the genesis block in April, 2014) Monero blocks were mined once per minute, but in March, 2016, it became two minutes per block [11]. To keep the 'emission schedule', i.e. the rate of money creation,[8] the same, block rewards were doubled. This just means,

[7] Monero amounts are stored in atomic-unit format in the blockchain.
[8] For an interesting comparison of Monero and Bitcoin's emission schedules see [15].

after the change, we use (L-M) >> 19 instead of >> 20 for new blocks. Currently the base block reward is

$$\mathrm{B} = \frac{(L - M) >> 19}{10^{12}}$$

6.3.2 Block size penalty

It would be nice to mine every new transaction into a block right away. However, what if someone submits a lot of transactions maliciously? The blockchain, storing every transaction, would quickly grow unpleasantly large.

One mitigation is a fixed block size, so the number of transactions per block is limited. What if honest transaction volume rises? Each transaction author would bid for a spot in new blocks by offering fees to miners. Miners would mine transactions with the highest fees in order to earn maximum money. As transaction volume increases, fees would become prohibitively large for transactions of small amounts (such as Alice buying an apple from Bob). Only people willing to outbid everyone else would get their transactions into the blockchain.

Monero avoids those extremes (limited vs unlimited block size) with a dynamic block size. Miners can make blocks bigger than typical blocks from the recent past, but they have to pay a penalty in the form of reduced block reward. It's the price of making a bigger block.

To calculate a new block's size penalty, we sample the most recent 100 blocks in the blockchain and find the *median* block size, *median_100blocks*. We set the variable M100 to the larger value in the set {*median_100blocks*, 300kB}.[9] Only blocks larger than M100 pay a penalty, but the median can slowly rise, allowing progressively bigger blocks with no penalty. The maximum block size is 2*M100.

If the intended block size is greater than M100, then, given base block reward B, the block reward penalty is

$$\mathrm{P} = \mathrm{B} * ((\text{block\_size}/\mathrm{M100}) - 1)^2$$

The actual block reward is therefore

src/cryptonote_basic/cryptonote_basic_impl.cpp get_block_reward()

$$\mathrm{B}^{\text{actual}} = \mathrm{B} - \mathrm{P}$$
$$\mathrm{B}^{\text{actual}} = \mathrm{B} * (1 - ((\text{block\_size}/\mathrm{M100}) - 1)^2)$$

Using the ˆ2 operation means penalties are sub-proportional to block size. A block size 10% larger than M100 has just a 1% penalty, and so on. [15]

We can expect miners to create blocks larger than M100 when the fee from adding another transaction is bigger than the penalty incurred.

[9] In the beginning of Monero it was 20kB, then increased to 60kB in March, 2016, (v2 of the protocol) [11], and has been 300kB since April, 2017 (v5 of the protocol) [1]. This non-zero 'floor' on the dynamic block size helps with transient transaction volume changes, especially in the early stages of Monero adoption.

6.3.3 Dynamic minimum fee

To prevent malicious actors from flooding the blockchain with transactions, which could be used to pollute ring signatures, and generally bloat it unnecessarily, Monero requires a minimum fee per kB of transaction data. Originally this was simply 0.002 XMR/kB, then in January, 2017, (v4 of the protocol) a formula was added to mitigate some security risks from transaction fees higher than block rewards, and to deincentivize miners from pushing M100 above 300kB during transient high volumes of submitted transactions [8].

First we say the base dynamic fee is $f_b^{kB} = 0.0004/\text{kB}$,[10] then we compute B^{actual} and M100 from the previous sections.

The minimum fee per kB is[11]

src/cryptonote_core blockchain.cpp get_dynamic_per_kb_fee()

$$f^{kB} = f_b^{kB} * (300\text{kB}/\text{M100}) * (\text{B}^{\text{actual}}/10)$$

Note: we round transaction size up to the nearest kB.

6.3.4 Emission tail

Let's suppose a cryptocurrency with fixed maximum supply and dynamic block size. After a while its block rewards fall to zero. With no more penalty on increasing block size, miners will simply add any transaction with a non-zero fee to their blocks.

Block sizes will stabilize around the average rate of transactions submitted to the network, and transaction authors will have no compelling reason to use transaction fees above the minimum, which would be zero according to Section 6.3.3.

This introduces an unstable, insecure situation. Miners will have little to no incentive to mine new blocks, leading to a fall in network hash rate as returns on investment decline. Block times will remain the same as difficulties adjust, but the cost of performing a double-spend attack may become feasible for malicious actors.

Monero mitigates this by not allowing the block reward to fall below 0.6 XMR (0.3 XMR per minute). This means when the following condition is met,

src/cryptonote_basic/ cryptonote_basic_impl.cpp get_block_reward()

$$\begin{aligned} 0.6 &> ((L - M) >> 19)/10^{12} \\ \text{M} &> \text{L} - 0.6 * 2^{19} * 10^{12} \\ \text{M}/10^{12} &> \text{L}/10^{12} - 0.6 * 2^{19} \\ \text{M}/10^{12} &> 18,132,171.273709551615 \end{aligned}$$

then the Monero chain will enter a so-called 'emission tail', with constant 0.6 XMR (0.3 XMR/minute) block rewards forever after.[12]

[10] The base fee was changed from 0.002 XMR/kB to 0.0004 XMR/kB in April, 2017 (v5 of the protocol) [1].

[11] To check if a given fee is correct, we allow a 2% buffer on f^{kB} in case of integer overflow (we compute fee before tx size is completely determined). This means the effective minimum fee is $0.98 * f^{kB}$.

[12] The Monero emission tail's estimated arrival is May, 2022 [13].

6.3.5 Miner transaction

Each block has a miner transaction that permits whoever mined a block to send himself the block reward and any transaction fees from transactions included in the block (summed into one output).[13] The output of a miner transaction is locked, unspendable, for 60 blocks after it is published [18].[14]

src/cryptonote_core/cryptonote_tx_utils.cpp construct_miner_tx()

Since RingCT was implemented in January, 2017 (v4 of the protocol) [12], people downloading a new copy of the blockchain compute a commitment to the miner transaction (a.k.a tx) amount a, as $C = 1G + aH$, and store it for referral. This means block miners can spend their miner transaction outputs just like a normal transaction's output, mixing them into MLSAG rings with other transaction outputs (both normal and miner tx's are eligible).

src/blockchain_db/blockchain_db.cpp add_transaction()

Blockchain verifiers store each post-RingCT block's miner tx amount commitment, for 32 bytes each, and compute them with:

PA	Point addition for some points A, B: $A + B$ [1]
KBSM	Known-base scalar multiplications for some integer a: aG [2]

6.4 Blockchain structure

The style of blockchain Monero uses is simple.

It starts with a genesis message of some kind (in our case basically a miner transaction dispersing the first block reward), which constitutes the genesis block. The next block contains a reference to the previous block, in the form of block ID. A block ID is simply a hash of: the block's header (a list of information about a block), a so-called 'Merkle root' that attaches all the block's transaction IDs (which are hashes of each transaction), and the number of transactions (including the miner transaction).

To produce a new block, one must do proof of work hashes by changing a nonce value stored in the block header until the difficulty target condition is met. The proof of work and block ID hash the same information, except use different hash functions.

src/cryptonote_basic/cryptonote_format_utils.cpp get_block_hashing_blob()

[13] The miner transaction output can theoretically be sent to a subaddress and/or use multisig and/or encoded (or not) payment ID. We don't know if any implementations have any of those features. Note that we need a transaction public key as usual.

[14] Any transaction's author can lock its outputs, rendering them unspendable until after a specified block height. He only has the option to lock all outputs to the same block height. It is not clear if this offers any meaningful utility to transaction authors. Locked outputs are only mandatory for miner transactions.

6.4.1 Transaction ID

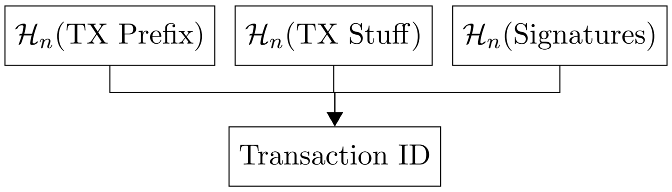
Transaction IDs are similar to the message signed by input MLSAG signatures (Section 5.6.3), but include the MLSAG signatures too.

The following information is hashed:

src/cryptonote_basic/cryptonote_format_utils.cpp calculate_transaction_hash()

- TX Prefix = {transaction era version (i.e. ringCT = 2), inputs {key offsets, key images}, outputs {one-time addresses}, extra {transaction public key, payment ID or encoded payment ID, misc.}}
- TX Stuff = {signature type (null/miner vs simple vs full), transaction fee, pseudo output commitments for inputs, ecdhInfo (masks and amounts), output commitments}
- Signatures = {MLSAGs, range proofs}

In this tree diagram, we use a black arrow to indicate a hash of inputs.


$\mathcal{H}_n$(TX Prefix)
$\mathcal{H}_n$(TX Stuff)
$\mathcal{H}_n$(Signatures)
Transaction ID

In place of an 'input', a miner transaction records the block height of its block. This ensures the miner transaction's ID, which is simply a normal transaction ID except with $\mathcal{H}_n(\text{Signatures}) \to \mathcal{H}_n(0)$, is always unique. No one can make different blocks with the same miner tx ID by setting the miner transaction's amount (i.e. with fees) and output one-time address the same, which could confuse people looking for the ID.

6.4.2 Merkle tree

Some users may want to discard unnecessary data from their copy of the blockchain. For example, once you verify some transaction's range proofs and input signatures, the only reason to keep that signature information is so users who obtain it from you can verify it for themselves.

To facilitate 'pruning' transaction data, and to more generally organize it within a block, we use a Merkle tree [47], which is just a binary hash tree of transaction IDs. Any branch in a Merkle tree can be pruned if you keep its root hash.[15]

src/crypto/tree-hash.c tree_hash()

[15] We do not know of any Monero users who prune their blockchain, nor of any software able to do it. If pruning *were* implemented, it would probably involve deleting all signature data after verification, and keeping $\mathcal{H}_n$(Signatures) for computing transaction IDs. Signatures constitute most of a block's data, so this could allow a substantial reduction in blockchain size.

An example Merkle tree based on four transactions and a miner transaction is diagrammed in Figure 6.1.[16]

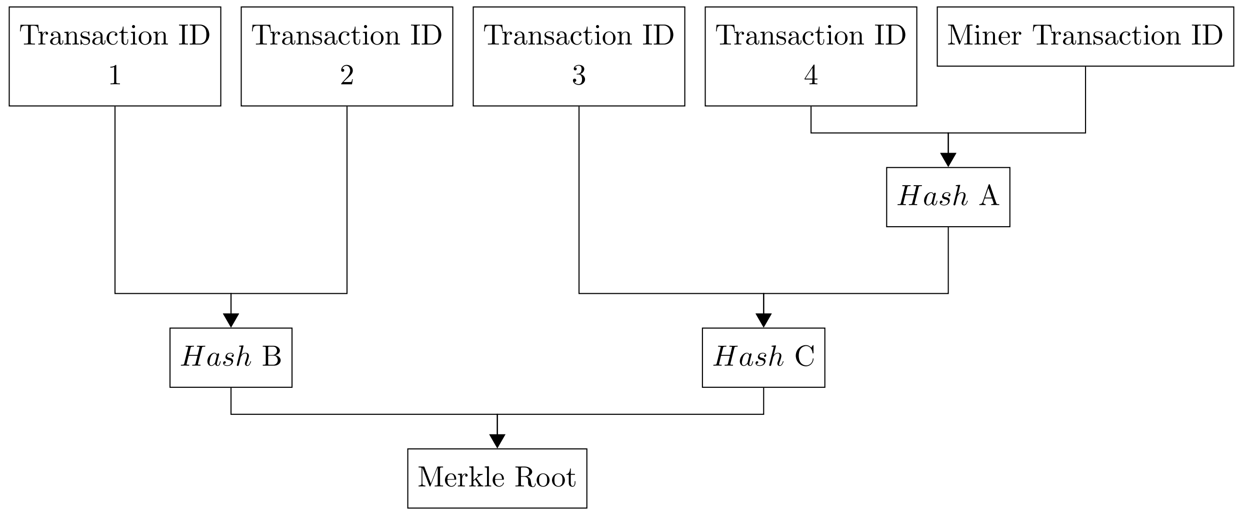


Figure 6.1: Merkle Tree

We can use the Merkle root to reference all of a blocks transactions, while facilitating transaction, or transaction component, pruning.

[16] A bug in the code for computing Merkle trees led to a serious attack on Monero on September 4, 2014 [38].

6.4.3 Blocks

A block is basically a block header and some transactions. Block headers record important information about each block. A block's transactions can be referenced with their Merkle root. We present here the outline of a block's content. Our readers can find a real block example in Appendix C.

- Block header:
  - **Major version**: Used to track hard forks (changes to protocol).
  - **Minor version**: Previously used for voting, now just displays the major version again.
  - **Timestamp**: UTC (Coordinated Universal Time) time of block. Added by miners, timestamps are unverified but they won't be accepted if lower than the median timestamp of the last 60 blocks.
  - **Previous block's ID**: Referencing the previous block, this is the essential feature of a blockchain.
  - **nonce**: A 32 byte integer that miners change over and over until the PoW hash meets the difficulty target. Block verifiers can easily recalculate the PoW hash.

src/cryptonote_core/blockchain.cpp check_block_timestamp()

- Miner transaction: Disperses block reward and transaction fees to the block's miner.
- Transaction IDs: References to non-miner transactions added to the blockchain by this block. Tx IDs can, in combination with the miner tx ID, be used to calculate the Merkle root, and to find the actual transactions wherever they are stored.

src/cryptonote_basic/cryptonote_format_utils.cpp calculate_block_hash() get_block_longhash()

Block IDs are computed like this:[17]

$$\text{Block ID} = \mathcal{H}_n(\text{Block header}, \text{Merkle root}, \#\text{transactions} + 1)$$

And block mining is performed like this:
- while $PoW_{output} > target$, keep changing the nonce and recalculating:

$$PoW_{output} = \mathcal{H}_{PoW}(\text{Block header}, \text{Merkle root}, \#\text{transactions} + 1)$$

In addition to the data in each transaction (Section 5.8), we store the following information:

- Major and minor versions: variable integers ≤ 64 bits
- Timestamp: variable integer ≤ 64 bits
- Previous block's ID: 32 bytes
- nonce: 32 bytes
- Miner transaction: 32 bytes for one-time address, 32 bytes for transaction public key, and variable integers for unlock time, height, and amount. After downloading the blockchain, we also need 32 bytes to store a commitment to post-RingCT miner tx amounts, $C = 1G + aH$, which are computed with:

PA Point addition for some points A, B: $A + B$ [1]

KBSM Known-base scalar multiplications for some integer a: aG [2]

- Transaction IDs: 32 bytes each

[17] +1 accounts for the miner tx.

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Appendices

APPENDIX A

RCTTypeFull Transaction Structure

We present in this chapter a dump from a real Monero transaction of type `RCTTypeFull`, together with explanatory notes for relevant fields.

The dump was obtained executing command `print_tx <TransactionID> +hex +json` in the `monerod` daemon run in non-detached mode. `<TransactionID>` is a hash of the transaction. The first line printed shows the actual command run.

To replicate our results, readers can do the following:[1]

1. We need the Monero command line tool (CLI), which can be found at `getmonero.org/downloads` (among other places). Get the 'Command Line Tools Only' for your operating system, move the file to a useful location, and unzip it.
2. Open the terminal/command line and navigate into the folder created by unziping.
3. Run `monerod` with `./monerod`. Now the Monero blockchain will download. Unfortunately, there is currently no easy way to print transactions without downloading the blockchain.
4. After the syncing process is done, commands like `print_tx` will work. Use `help` to learn other commands.

For editorial reasons we have shortened long hexadecimal chains, presenting only the beginning and end as in `0200010c7f[...]409`.

[1] Blockchain data can also be found with a web block explorer, such as `https://moneroblocks.info/`.

Component `rctsig_prunable`, as indicated by its name, is in theory *prunable* from the blockchain. That is, once a block has been consensuated and the current chain length rules out all possibilities of double-spending attacks, this whole field could be pruned and replaced with its hash for the Merkle tree. This is something that has not yet been done in the Monero blockchain, but is nevertheless a possibility, and would yield considerable space savings.

Key images and ring keys are stored separately, in the non-prunable area of transactions. These components are essential for detecting double-spend attacks and can't be pruned away.

Our sample transaction has 1 input and 2 outputs, and was added to the blockchain at timestamp 2017-12-18 20:28:11 UTC (as reported by its block's miner).

```
print_tx  b43a7ac21e1b60ad748ec905d6e03cf3165e5d8c9e1c61c263d328118c42eaa6 +hex +json
Found in blockchain at height 1467685
0200010c7f[...]409
{
   "version": 2,
   "unlock_time": 0,
   "vin": [ {
      "key": {
         "amount": 0,
         "key_offsets": [ 799048, 782511, 1197717, 216704, 841722
         ],
         "k_image": "595a612d0df27181c46a8af70a9bd682f2a000124b873ba5d2b9f4b4e4efd672"
      }
   }
   ],
   "vout": [ {
      "amount": 0,
      "target": {
         "key": "aa9595f55f2cfaed3bd2a67453bb064dc7fd454a09c2418d7338782790185fe3"
      }
   }, {
      "amount": 0,
      "target": {
         "key": "0ccb48ed2ebbcaa8e8831111029f3300069cff0d1408acffbfc3810b362ea217"
      }
   }
   ],
   "extra": [ 2, 33, 0, 129, 70, 77, 194, 248, 93, 24, 94, 15, 107, 233, 0, 229, 82,
   175, 243, 123, 58, 204, 135, 171, 100, 101, 192, 42, 187, 157, 168, 222, 98, 192,
   110, 1, 1, 185, 87, 22, 38, 116, 81, 124, 85, 68, 36, 44, 229, 235, 46, 159, 139,
   114, 234, 211, 50, 41, 28, 92, 26, 249, 184, 228, 197, 64, 139, 5
   ],
   "rct_signatures": {
      "type": 1,
      "txnFee": 26000000000,
      "ecdhInfo": [ {
         "mask": "68f508c5515694ce5a33b316b990e8b67a944725c93d806767e61b2e0b13d300",
         "amount": "913372a2424b22bd9712183f5a7c8027c8d9af89b52d1e7d06fd1f87a1e5d20d"
      }, {
         "mask": "fbc3e5bdb36fc58e5800ffc549ab7bd533fadb7e6b64898c82ea620d749fc80e",
         "amount": "b9335c3dc0afb774f812f9f58a412c849f3c828d873f1c16ab102963799d9809"
      }],
      "outPk": [ "cf141f5dfe04df14afad6b451d600aa5826a9be44a76a1630850c1d5951d482e",
                 "e10bb69b66af5dabec765c7f5f7528926088877fa36746833828a0575896ae57"]
   },
   "rctsig_prunable": {
      "rangeSigs": [ {
         "asig": "b9b544a7[...]d4c5726e81c4c4b6205dacc05208",
         "Ci": "bc7ae457[...]fe490458"
      }, {
         "asig": "9c457b41[...]545b60c",
         "Ci": "ce9b4d8e[...]03a6752"
      }],
      "MGs": [ {
         "ss": [[ "a8120b96f5f2ac5bceab37f7d6bf8d86554d87c4af3441007cad92f54a24d908",
          "2e6bc016297a5d398936c9f45e7a80215138f69e55179b337922e2d51c1a9f00"],
          ["1e1052a68c38bb88b6e8f257d999c13f1d5f4fa219cc23479ccbfa6b14b5960a",
           "e914d35eed0d27344fbc3a89b91bd445d433b561efc844c9f466a61ebb5f6d09"],
          ["e04d011f515461fdbd8d13536c23143dc365d87dd323defb1af834e540a8fc0e",
           "f9b41a117a1415fec54f1cc16aeef859b2cab1494b9e26a95fc9eaf4f571fa00"],
          ["de7a7b30795cab310b632f708c6c2546847a5cbcc27ff48e75c1556c3f6f180c",
           "6218695558359d115e308b008d9aa368c38672732d2fc21c6317ad7d15918c05"],
          ["0ca70bbdea0e391b1e24e2540f33b48dd9dc554c61ebf23bb3691aab5094e40f",
           "dafecd436b2448504c0a3a1997b356c141f1d4b5977cc66e5f55592f13731501"]],
         "cc": "5059757cf06216215955aaa108e8dd40be157856749a9d883bcac611e395a409"
      }]
   }
}
```

Transaction components

- (line 2) - `print_tx` reports where it found the transaction.
- (line 3) - Raw transaction data with no readability formatting.
- `version` (line 5) - Transaction format/era version; '2' corresponds to RingCT.
- `unlock_time` (line 6) - Prevents a transaction's outputs from being spent until the time has past. It is either a block height, or a UNIX timestamp if the number is larger than the beginning of UNIX time.
- `vin` (line 7-14) - List of inputs (there's only one here)
- `amount` (line 9) - Deprecated (legacy) amount field for type 1 transactions
- `key_offset` (line 10) - This allows verifiers to find ring member keys and commitments in the blockchain, and makes it obvious those members are legitimate. The first offset is absolute within the blockchain history, and each subsequent offset is relative to the previous. For example, with real offsets {7,11,15,20}, the blockchain records {7,4,4,5}. Verifiers compute the last offset like (7+4+4+5 = 20) (Section 5.6.6).
- `k_image` (line 12) - Key image $\tilde{K}_j$ from Section 3.3, where $m = j = 1$ for the 1 input here.
- `vout` (lines 16-27) - List of outputs (there are two here)
- `amount` (line 17) - Deprecated amount field for type 1 transactions
- `key` (line 19) - One-time destination key for output t as described in Section 5.2
- `extra` (lines 28-32) - Miscellaneous data, including the *transaction public key*, i.e. the share secret rG of Section 5.2, and payment ID or encoded payment ID from Section 5.4. It works like this: each number is one byte (it can be 0-255), and each kind of thing that can be in the field has a 'tag' and 'size'. Tag indicates which information comes next, and size indicates how many bytes that info occupies. The first number is always a tag. Here, '2' indicates an 'extra nonce' which is useful for mining pools, and then '33' means the nonce has 33 bytes. 33 numbers go by, then we find a new tag, '1', which means 'transaction public key'. Tx public keys are always 32 bytes, so we don't need to include the size. 32 numbers later is the end of this extra field. (note: in original specification the first byte indicated the size of the field. Monero doesn't use that.) [23]

  src/cryptonote_basic/tx_extra.h
- `rct_signatures` (lines 33-45) - First part of signature data
- `type` (line 34) - Signature type; `RCTTypeFull` is type 1. `RCTTypeSimple` = 2, `RCTTypeNull` (miner tx) = 0.
- `txnFee` (line 35) - Transaction fee in clear, in this case 0.026 XMR

- `ecdhInfo` (lines 36-42) - 'elliptic curve diffie-hellman info': Obscured mask and amount for each of the outputs $t \in \{1, ..., p\}$; here $p = 2$
- `mask` (line 37) - Field *mask* at $t = 1$ as described in Section 5.6.1
- `amount` (line 38) - Field *amount* at $t = 1$ as described in Section 5.6.1
- `outPk` (lines 43-44) - Commitments for each output, Section 5.6.2
- `rctsig_prunable` (lines 46-67) - Second part of signature data
- `rangeSigs` (lines 47-53) - Range proofs for output $t \in 1, ..., p$ commitments
- `asig` (line 48) - List of Borromean signature terms (shortened for brevity) for the range proof of output $t = 1$ (includes all c and r), see Section 5.6.6
- `Ci` (line 49) - List of commitments (shortened for brevity) for the range proof on output $t = 1$, as described in Section 5.6.3. As explained in Section 5.6.6 only the commitments C_i need to be stored, as the values $C_i - 2^i H$ can be easily derived by verifiers.
- `MGs` (lines 54-66) - Remaining elements of the MLSAG signature
- `ss` (lines 55-64) - Components $r_{i,j}$ from the MLSAG signature
$$\sigma(\mathfrak{m}) = (c_1, r_{1,1}, ..., r_{1,m+1}, ..., r_{v+1,1}, ..., r_{v+1,m+1})$$
- `cc` (line 65) - Component c_1 from aforementioned MLSAG signature

APPENDIX B

RCTTypeSimple Transaction Structure

In this section we show the structure of a sample transaction of type `RCTTypeSimple`. The transaction has 4 inputs and 2 outputs, and was added to the blockchain at timestamp 2017-12-21 11:36:20 UTC (as reported by its block's miner).

```
print_tx 3ebf45fc5f8fd6830378073841228l7d5debfa762c7a7845cb7ccfe9ee20940b
Found in blockchain at height 1469563
020004[...]923b3d70d
{
  "version": 2,
  "unlock_time": 0,
  "vin": [ {
      "key": {
        "amount": 0,
        "key_offsets": [ 1567249, 1991110, 349235, 15551, 3620
        ],
        "k_image": "9661119b4b54529e1be14ef97fbdc0504d17a6c8dfedd55d2455b93a6336bb41"
      }
    }, {
      "key": {
        "amount": 0,
        "key_offsets": [ 2502375, 650851, 337433, 396459, 39529
        ],
        "k_image": "2102414d8edfa229f9ebf32ab90acd9cf23963a8c3b6ba0e181fc1d5782c046c"
      }
    }, {
      "key": {
        "amount": 0,
        "key_offsets": [ 1907097, 696508, 806254, 510195, 6709
        ],
        "k_image": "de14ec8958b311bd38a05aa3fb08fdd360001f1b9c060264eecdd8c08c9e83c4"
      }
    }, {
      "key": {
        "amount": 0,
        "key_offsets": [ 1150236, 1943388, 788506, 37175, 7462
        ],
        "k_image": "e470f77dd5a4149210cb61ee107e73caea1ef9f61d05384e3bd4372fdc85bf17"
      }
    }
  ],
  "vout": [ {
      "amount": 0,
      "target": {
        "key": "787cad1ebb181e1fc04b24d4d06c3d2882c38b262a7635de8ad487c536e40a12"
      }
    }, {
      "amount": 0,
      "target": {
        "key": "faf4137928392b39ccf0a830c0261573009959787697f9d4fb769c25781fb911"
      }
    }
  ],
  "extra": [ 1, 20, 56, 120, 111, 89, 89, 64, 10, 98, 96, 255, 202, 235, 203,
             255, 2, 197, 176, 147, 61, 60, 41, 145, 207, 178, 212, 71, 37, 69, 19,
             147, 205],
  "rct_signatures": {
    "type": 2,
    "txnFee": 558805800000,
    "pseudoOuts": [ "64dea29ac5560f93773240d58ca5768b879fd3c95e0b3b50a80ec36a6ff3a6da",
                    "a60e7a00e65ff2a6299b92b166a629e9b0d62f6df50e40535140716757efe4c0",
                    "4c67403adbc9dc0ca5a1a6abc846ab6d232dc3fa295099b3c7a9d005bac60eba",
                    "635b26d78117d77899859ecb61e10125c3956a5c113b932f33c92c561acddaa3"],
    "ecdhInfo": [ {
        "mask": "ccffac42a86bec7b36ce9957cbdfe481d419bc5353335d0c236c347aea758d0c",
        "amount": "c0d6cf3e1db55dd459b73faf34d7339c3fa1b3d3356cfb2adc3faf798264b00e"
      }, {
        "mask": "62cc846003d9c5425c6cf33b30754a4c044f5d9d02621460e45664b886673109",
        "amount": "726dbacad62022bf0f5af05c72482b3f040d631d3f576b5e2615ea72f84c5f06"
      }],
    "outPk": [ "cb3b729b4fca6e66736666201633e3f905c367a2f3d18e31fe3d3c18d2be93fd",
               "1e8c86b7f211a99e1762bf62254efe65ea5c5328b62b0ea8d679b2e52800f633"]
  },
  "rctsig_prunable": {
    "rangeSigs": [ {
        "asig": "9bb7cce09[...]61de7ce0a",
        "Ci": "50dfd2e8[...]b3a7c8fca1b1"
      }, {
        "asig": "e3905fa5c[...]b5213444908",
        "Ci": "595c2cec5f2[...]72a628ab5c"
      }],
    "MGs": [ {
        "ss": [ [ "3b2d26ea7628015fd8317e4e298ceda6b534ac894b83f7b6190a353cee6ec702",
                  "2d772ad7b7ff2ba8a1a66c8d69c0a0d49d72808eaf803c59f13c3d78b653440c"],
                [ "c188891fb37d76305f0209222f52d22ede43018facfe91f949ecb8dcf709b30a",
                  "303896ca67ea7969544641d5bb94a436558bcf6522bb9bc77bd1abb5f2146c08"],
                [ "e01c88b7308403a9dd023d9eacf1ade17ab0fa54250148431b5a33c98e636100",
                  "ba36e34e5245e89c7c21af845b949cf3e82188df639390f094e31c9ba773060c"],
                [ "37175d72d2bef3f8bd9e65fb2861f7bce91e3b1e30278b2dcf26112831ac9405",
                  "8e77a5dd641a89e86dbe0708e8f59d0e2dc9fe4ddfd9b367c3a93522198a4706"],
                [ "fbb4f94f9ce0f081421e63677a63d5914f0536a481d57b6e5fc5379c84dfcb05",
                  "1ec40aa3c8a94c6b1915b7796423b0d7d6011aa2d6af636aff309b832f193408"]],
        "cc": "a03119e4257cca37f89ac3e97f0598b712c79162c73932d58ab4ce08c4ad6709"
      }, {
        "ss": [ [ "f7aedeec462d7588330c71589fde5f0f234a627a6e5ed72cff34825a04d41707",
                  "31d7a5ba4e782db5c0704ab751a2ef8c4732f3cf699bc8f9994e79a97cd3190e"],
                [ "00e1d1ecdf31fdf7d57661f2234bfc859cfdc4dbfdfd0f5eec0576ef22592203",
                  "fdf08b803fa6de18bf0e0dc6855e877877bda0101eceb81e2223fe0175606300"],
                [ "3397ba3f9e8db066e3c4911b896debefbc73efbac4988e6aff5731ff8db15405",
                  "43d2b03d5263de99f56c256e646be503edd63dd03d377a469379fbf487e8600e"],
                [ "31afd1d5c3b07170ac127605fc35cbcc19cf963a35b2ff8f804e17e3b804000d",
                  "3a3bc124a10b0cf416656f8f682a427445140895440cca644c6aa38966399f0c"],
                [ "f499f0e4922d5cba35e3cc033489b60ec7ea26ff19cc9dd29357670f4bf8790b",
                  "1a4732b31f0f1a7d3322be5c4baca098f0a032c192bf9f8a6b5fd83cbdd9d401"]],
        "cc": "095fcab7ebf64c2ffecdacf11b70f97f0e709de0a84b3a13abca627f9df2c901"
      }, {
        "ss": [ [ "1808924b4154118c48f0b305562b6ffba86f38c64d4d8a087823f3383cddd006",
                  "4b18544be50aee8c4594b568d6be741c155a132cb83392d9b1a4cf35c3d5760c"],
104                  [ "9a95eeeecf3c3a48c43873a372c263357ff5f258a7bf8ed29a767237b0b0f202",
105                    "4f1ea4d9d4b56db780dd078a3e8219d0f54eaccc197901671002a206f063cb0e"],
106                  [ "2c800017cab2b8388f58fed0d61a46570f64cacd8fabc4e84ddee735b3135f0a",
107                    "458016f6fdf58b329fae0f929226ee2b8e410a14db8c6ede9b74fb718de71507"],
108                  [ "ea0fcaf793602dce25c8b2c4d17163f4298933b3fb09874307d8cde9a63c2c0c",
109                    "df76fbcd336c07f37e90e1a0d0db1ba49519ba4325062228bc9242af2c525703"],
110                  [ "e3a7a0477eeb602a9a8203a6a496cca90c4769d57410246c4c8d665df34df900",
111                    "44d206154f0ca85e12a92eefdbc3784e17e701a32ff93b550467679f67500c0d"]],
112          "cc": "6f80de1c1d566776d2831f15c9a85fb1d8e8cecfd0d2753b318f0e84d89d3b08"
113        }, {
114          "ss": [ [ "5b82b4644b57e3d623de7c72c6ebd52959815c12c80b479e4cbe5437cf67640c",
115                    "b70e0b69c75faba6a1630429f9f497db351347c210467f69e1b1c5f1a72afe02"],
116                  [ "f25a93a98f980cf489eb8f69369f4ec63eaea91fd677decab9b6ca0fe2feb606",
117                    "124536c374cb6023a6aa6f22ae6e115a1ba12cb36c48f5f5ad43ce90f471da02"],
118                  [ "151ddc82322456d7f31b8b4b2290098c3bf2428370c7ef325660b5463ff26404",
119                    "2fb9d2979e16c2b1131686bb85068ec559f9c6c64581e609b451bb2cd9d5740d"],
120                  [ "c03bed01d6ad60b3da5d2c88cf2e5023b51133c37e4917511715a11f09d8740d",
121                    "432e01c2075ab6361af8636cc1c9254e12db98f5c323088792dfb42a1c894401"],
122                  [ "52206d801214e70d20ee7ca53823c143aa06c3d1b22b118cc8a15c9f861f0102",
123                    "563c134f56f7a290e0980877e93bc4b08651e53dade079b1e6c066b70fb81406"]],
124          "cc": "4102cd245db3e0d7c0e2280cfdba38b9b7a7ad8715b8fe68c1170cf923b3d70d"
125        }]
126    }
127  }
128
129
130
```

Transaction components

What follows is a short explanation of the most important elements in this type of transaction. We only mention components that are specific to, or differ from, the previous `RCTTypeFull` transaction type.

- `type` (line 53) - Signature type, in this case the value 2, corresponding to `RCTTypeSimple` transactions
- `pseudoOuts` (lines 55-58) - Pseudo-output commitments C'^{a}_{j}, as described in Section 5.7.1. Please recall that the sum of these commitments will equal the sum of the 2 output commitments of this transaction (plus the transaction fee commitment fH).

APPENDIX C

Block Content

In this section we show the structure of a sample block, namely the 1582196th block after the genesis block. The block has 5 transactions, and was added to the blockchain at timestamp 2018-05-27 21:56:01 UTC (as reported by the block's miner).

```
print_block 1582196
timestamp: 1527458161
previous hash: 30bb9b475a08f2ea6fe07a1fd591ea209a7f633a400b2673b8835a975348b0eb
nonce: 2147489363
is orphan: 0
height: 1582196
depth: 2
hash: 50c8e5e51453c2ab85ef99d817e166540b40ef5fd2ed15ebc863091ca2a04594
difficulty: 51333809600
reward: 4634817937431
{
  "major_version": 7,
  "minor_version": 7,
  "timestamp": 1527458161,
  "prev_id": "30bb9b475a08f2ea6fe07a1fd591ea209a7f633a400b2673b8835a975348b0eb",
  "nonce": 2147489363,
  "miner_tx": {
    "version": 2,
    "unlock_time": 1582256,
20      "vin": [ {
21          "gen": {
22            "height": 1582196
23          }
24        }
25      ],
26      "vout": [ {
27          "amount": 4634817937431,
28          "target": {
29            "key": "39abd5f1c13dc6644d1c43db68691996bb3cd4a8619a37a227667cf2bf055401"
30          }
31        }
32      ],
33      "extra": [ 1, 89, 148, 148, 232, 110, 49, 77, 175, 158, 102, 45, 72, 201, 193,
34      18, 142, 202, 224, 47, 73, 31, 207, 236, 251, 94, 179, 190, 71, 72, 251, 110,
35      134, 2, 8, 1, 242, 62, 180, 82, 253, 252, 0
36      ],
37      "rct_signatures": {
38        "type": 0
39      }
40    },
41    "tx_hashes": [ "e9620db41b6b4e9ee675f7bfdeb9b9774b92aca0c53219247b8f8c7aecf773ae",
42                   "6d31593cd5680b849390f09d7ae70527653abb67d8e7fdca9e0154e5712591bf",
43                   "329e9c0caf6c32b0b7bf60d1c03655156bf33c0b09b6a39889c2ff9a24e94a54",
44                   "447c77a67adeb5dbf402183bc79201d6d6c2f65841ce95cf03621da5a6bffefc",
45                   "90a698b0db89bbb0704a4ffa4179dc149f8f8d01269a85f46ccd7f0007167ee4"
46    ]
47  }
```

Block components

- (lines 2-10) - Block information collected by software, not actually part of the block properly speaking.
- `is orphan` (line 5) - Signifies if this block was orphaned. Nodes usually store all branches during a fork situation, and discard unnecessary branches when a cumulative difficulty winner emerges, thereby orphaning the blocks.
- `depth` (line 7) - In a blockchain copy, the depth of any given block is how far back in the chain it is relative to the most recent block.
- `hash` (line 8) - This block's block ID.

- `difficulty` (line 9) - Difficulty isn't stored in a block, since users can compute *all* block difficulties from block timestamps and the rules in Section 6.2.
- `major_version` (line 12) - Corresponds to protocol version used to verify this block.
- `minor_version` (line 13) - Originally intended as a voting mechanism among miners, it now merely reiterates the `major_version`. Since the field does not occupy much space, the developers probably thought deprecating it would not be worth the effort.
- `timestamp` (line 14) - An integer representation of this block's UTC timestamp, as reported by the block's miner.
- `prev_id` (line 15) - The previous block's block ID. Herein lies the essence of Monero's blockchain.
- `nonce` (line 16) - The nonce used by this block's miner to pass its difficulty target. Anyone can recompute the proof of work and verify the nonce is valid.
- `miner_tx` (lines 17-40) - This block's miner transaction.
- `version` (line 18) - Transaction format/era version; '2' corresponds to RingCT.
- `unlock_time` (line 19) - The miner transaction's output can't be spent until 60 more blocks have been mined.
- `vin` (lines 20-25) - Inputs to the miner tx. There are none, since the miner tx is used to generate block rewards and collect transaction fees.
- `gen` (line 21) - Short for 'generate'.
- `height` (line 22) - The block height this miner tx's block reward was generated for. Each block height can only generate a block reward once.
- `vout` (lines 26-32) - Outputs of the miner tx.
- `amount` (line 27) - Amount dispersed by the miner tx, containing block reward and fees from this block's transactions. Recorded in atomic units.
- `key` (line 29) - One-time address assigning ownership of the miner tx's amount.
- `extra` (lines 33-36) - Extra information for the miner tx, including transaction public key.
- `type` (line 38) - Type of transaction, in this case '0' for `RCTTypeNull`, indicating a miner tx.
- `tx_hashes` (line 41) - All transaction IDs included in this block (but not the miner tx ID).

APPENDIX D

Genesis Block

In this section we show the structure of the Monero genesis block. The block has 0 transactions (it just sends the first block reward to thankful_for_today [63]). Monero's founder did not add a timestamp, perhaps as a relic of Bytecoin, the coin Monero's code was forked from, whose creators apparently tried to hide a large pre-mine [10].

Block 1 was added to the blockchain at timestamp 2014-04-18 10:49:53 UTC (as reported by the block's miner), so we can assume the genesis block was created the same day. This corresponds with the launch date announced by thankful_for_today [63].

```
print_block 0
timestamp: 0
previous hash: 0000000000000000000000000000000000000000000000000000000000000000
nonce: 10000
is orphan: 0
height: 0
depth: 1580975
hash: 418015bb9ae982a1975da7d79277c2705727a56894ba0fb246adaabb1f4632e3
difficulty: 1
reward: 17592186044415
{
  "major_version": 1,
  "minor_version": 0,
  "timestamp": 0,
15   "prev_id": "0000000000000000000000000000000000000000000000000000000000000000",
16   "nonce": 10000,
17   "miner_tx": {
18     "version": 1,
19     "unlock_time": 60,
20     "vin": [ {
21         "gen": {
22           "height": 0
23         }
24       }
25     ],
26     "vout": [ {
27         "amount": 17592186044415,
28         "target": {
29           "key": "9b2e4c0281c0b02e7c53291a94d1d0cbff8883f8024f5142ee494ffbbd088071"
30         }
31       }
32     ],
33     "extra": [ 1, 119, 103, 170, 252, 222, 155, 224, 13, 207, 208, 152, 113, 94, 188,
34     247, 244, 16, 218, 235, 197, 130, 253, 166, 157, 36, 162, 142, 157, 11, 200, 144,
35     209
36     ],
37     "signatures": [ ]
38   },
39   "tx_hashes": [ ]
40 }
```

Genesis block components

Since we used the same software to print the genesis block and the block from Appendix C, the structure appears basically the same. We point out some unique differences.

- `difficulty` (line 9) - The genesis block's difficulty is reported as 1, which means any nonce could work.
- `timestamp` (line 14) - The genesis block doesn't have a meaningful timestamp.
- `prev_id` (line 15) - We use an empty 32 bytes for the previous ID, by convention.
- `nonce` (line 16) - $n = 10000$ by convention.
- `amount` (line 27) - This exactly corresponds to our calculation for the first block reward (17.592186044415 XMR) in Section 6.3.1.

- `key` (line 29) - The very first Moneroj were dispersed to Monero's founder thankful_for_today.
- `extra` (line 33) - Using the encoding discussed in Appendix A, the genesis block's miner tx `extra` field just contains one transaction public key.
- `signatures` (line 37) - There are no signatures in the genesis block. This is here as an artifact of the `print_block` function. The same is true for `tx_hashes` in line 38.